UNIVERSITÉ DE GENÈVE
FNSNF SWISS NATIONAL SCIENCE FOUNDATION

CROSS CORRELATIONS BETWEEN EUCLID AND CMB
PART 2 : CMB LENSING

EUCLID ADVANCED SCHOOL 2022

LOUIS LEGRAND

CMB E modes lensed - unlensed

SOME REVIEWS

- Lewis and Challinor 2006 https://arxiv.org/abs/astro-ph/0601594
- Hanson, Challinor and Lewis 2009 https://arxiv.org/pdf/0911.0612.pdf

OUTLINE

- What is the gravitational lensing of the CMB ?
- How do we measure it?
- Cross correlations with galaxy surveys
- Galaxy clusters and CMB lensing (optional)
- Next generation estimators of CMB lensing (optional)

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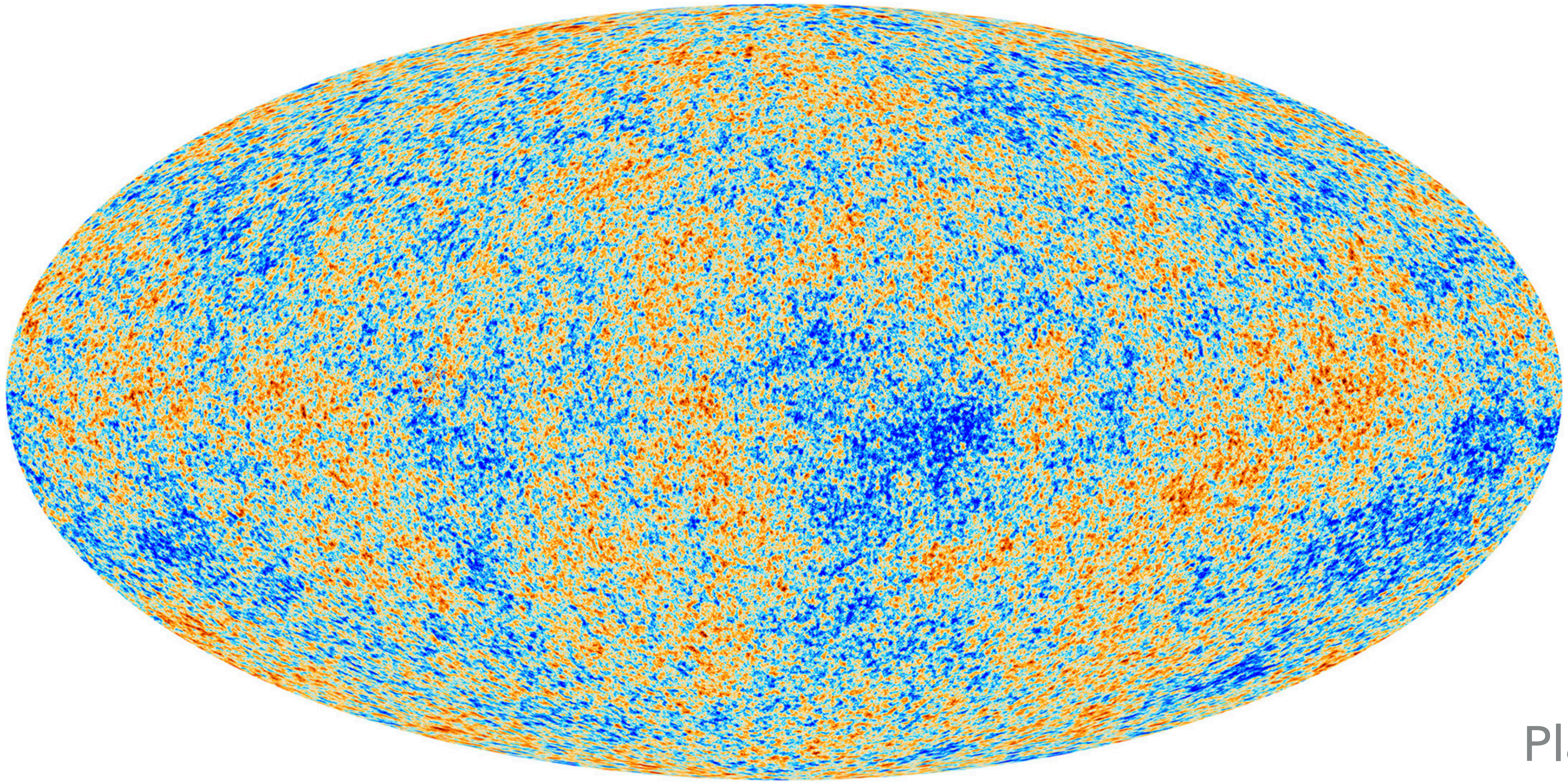
THE COSMIC MICROWAVE BACKGROUND

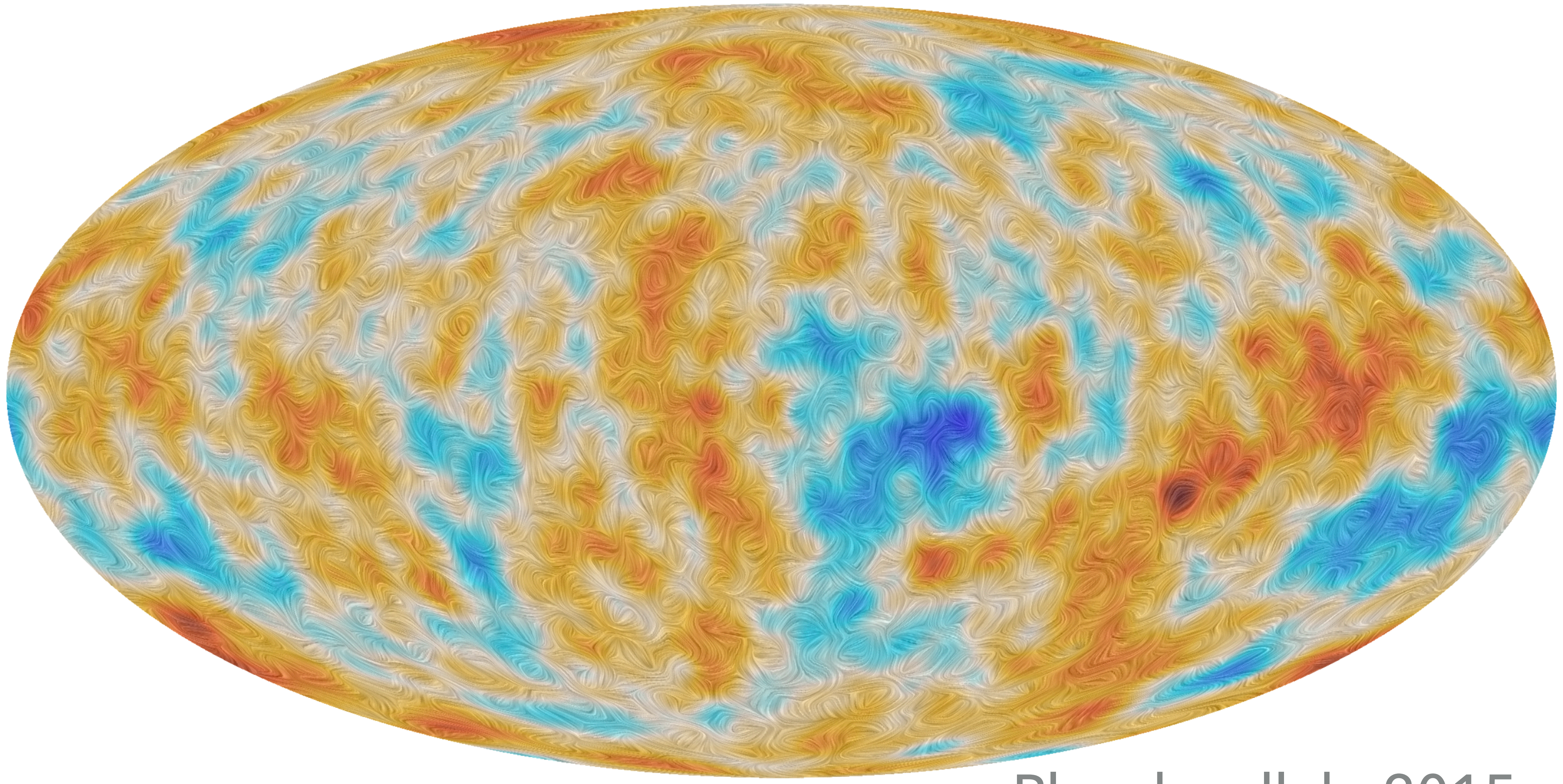


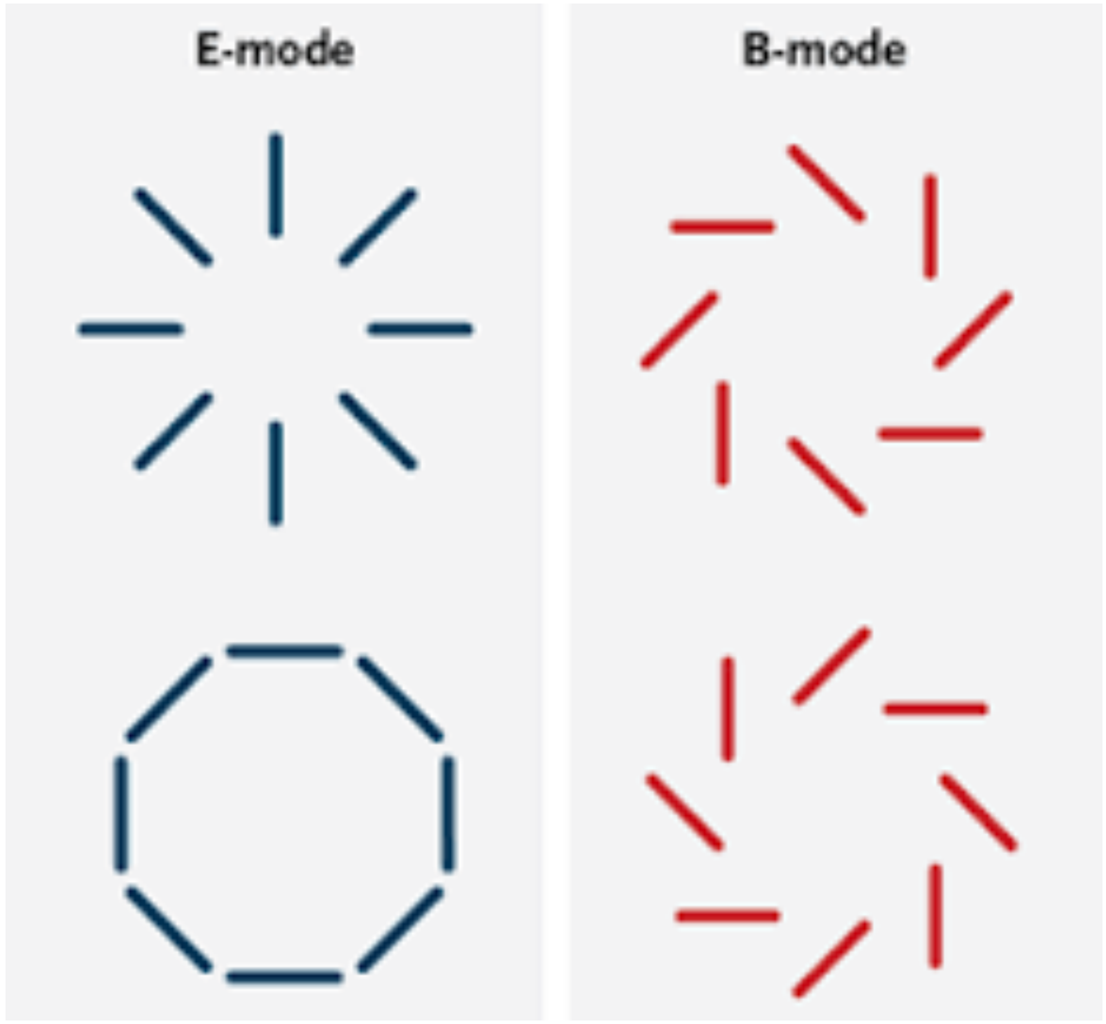
Planck collaboration 2013

- Perfect black body at 2.725 K with very tiny $O(10^{-5})$ anisotropies
- These perturbations are sourced by the initial power spectrum from inflation
- They have evolved under competing effects of gravity and pressure, while matter and photons were coupled

POLARISATION OF THE CMB

- CMB photons are polarised by perturbations of the electron density at recombination
- We decompose polarisation into E and B modes (analogy with electro-magnetism)


Planck collab. 2015



Primordial E modes:

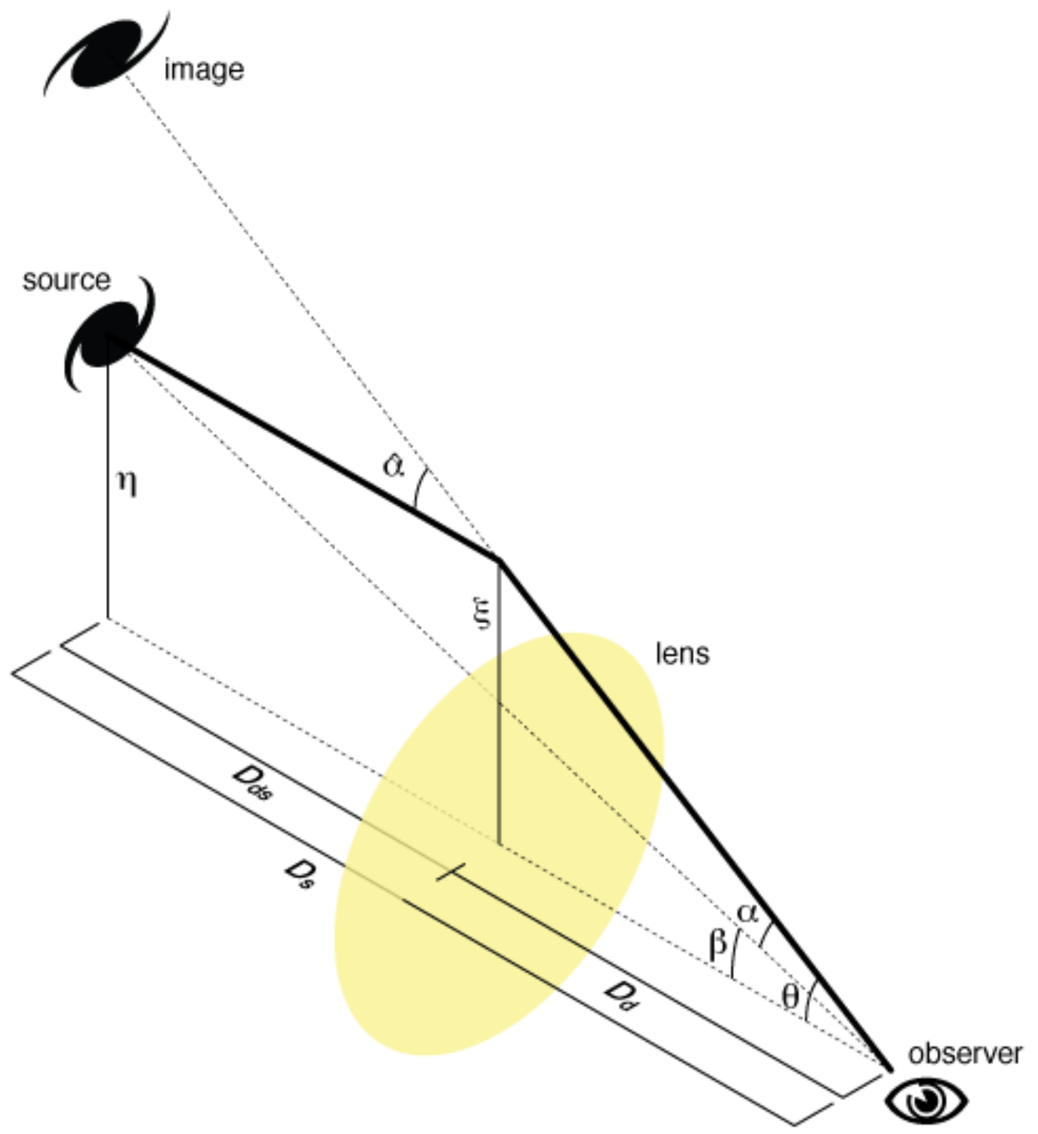
- Sourced by scalar and tensor perturbations

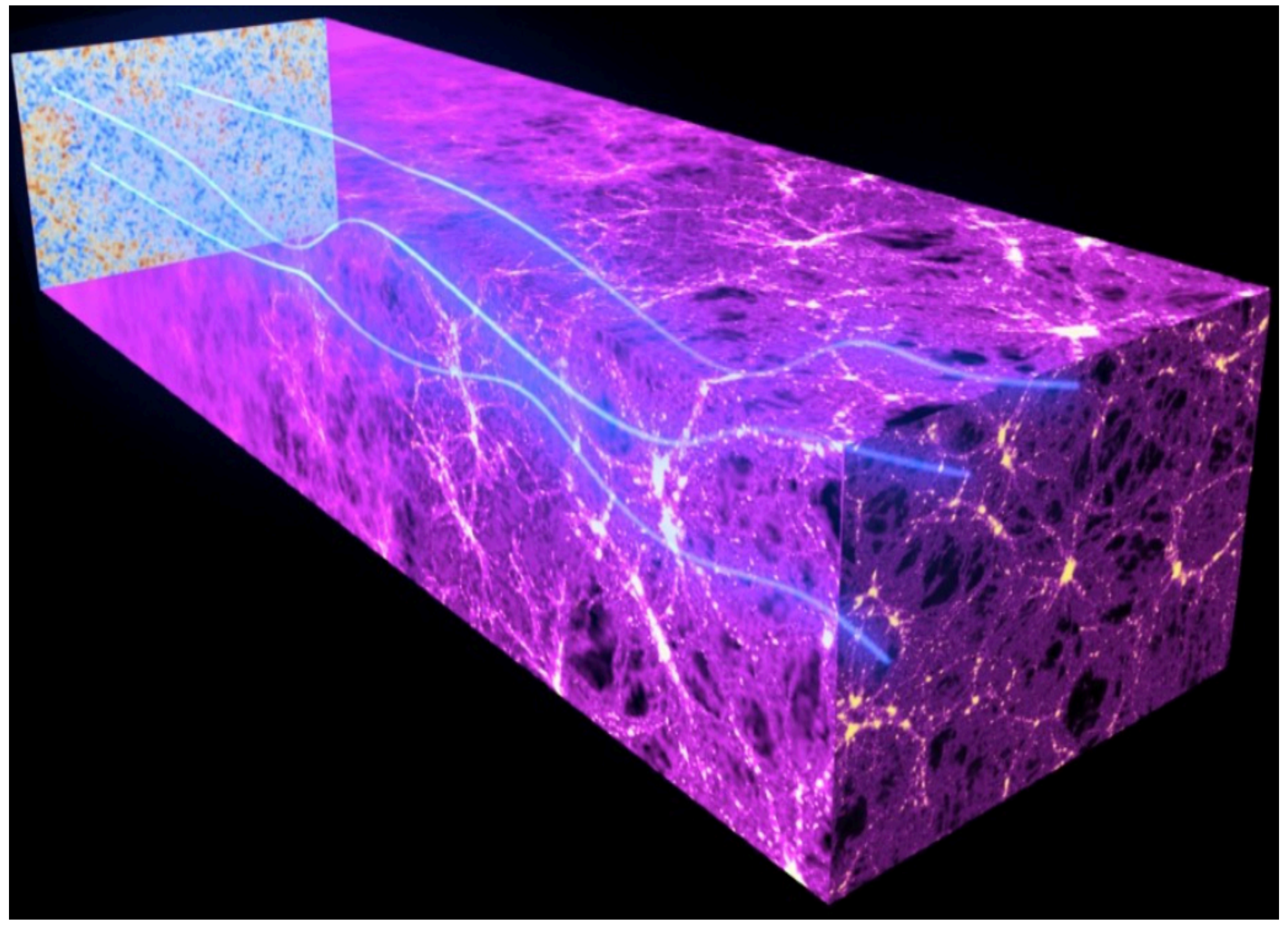
Primordial B modes:

- Sourced by tensor modes only = gravitational waves
- Primordial GW are relics of inflation
- But signal is dominated by secondary B modes from lensing

GRAVITATIONAL LENSING

- Photons are deflected by the mass along their trajectory
- Point source deflection angle is $\delta\theta = \dfrac{4MG}{c^2 b}$





- CMB act as a extended source at z=1100
- CMB photons are lensed by the large scale structures created by gravitational evolution of matter

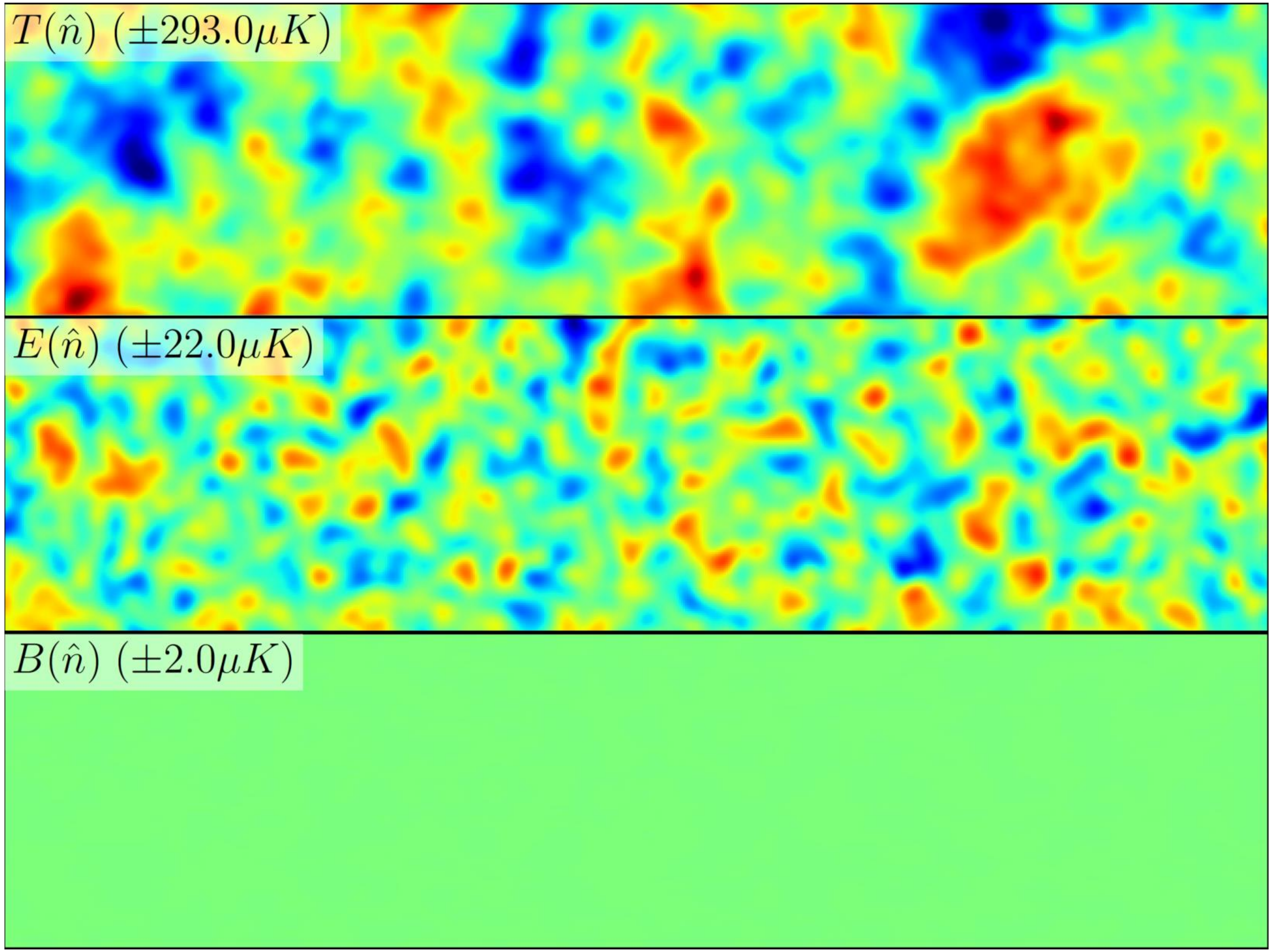
CMB LENSING

- Lensing acts as a remapping of the primordial CMB fields

$$X^{\mathrm{len}}(\boldsymbol{n}) = X^{\mathrm{unl}}(\boldsymbol{n} + \boldsymbol{\alpha}(\boldsymbol{n}))$$

- It creates statistical anisotropies and correlation between different scales

Blanchard and Schneider 1987



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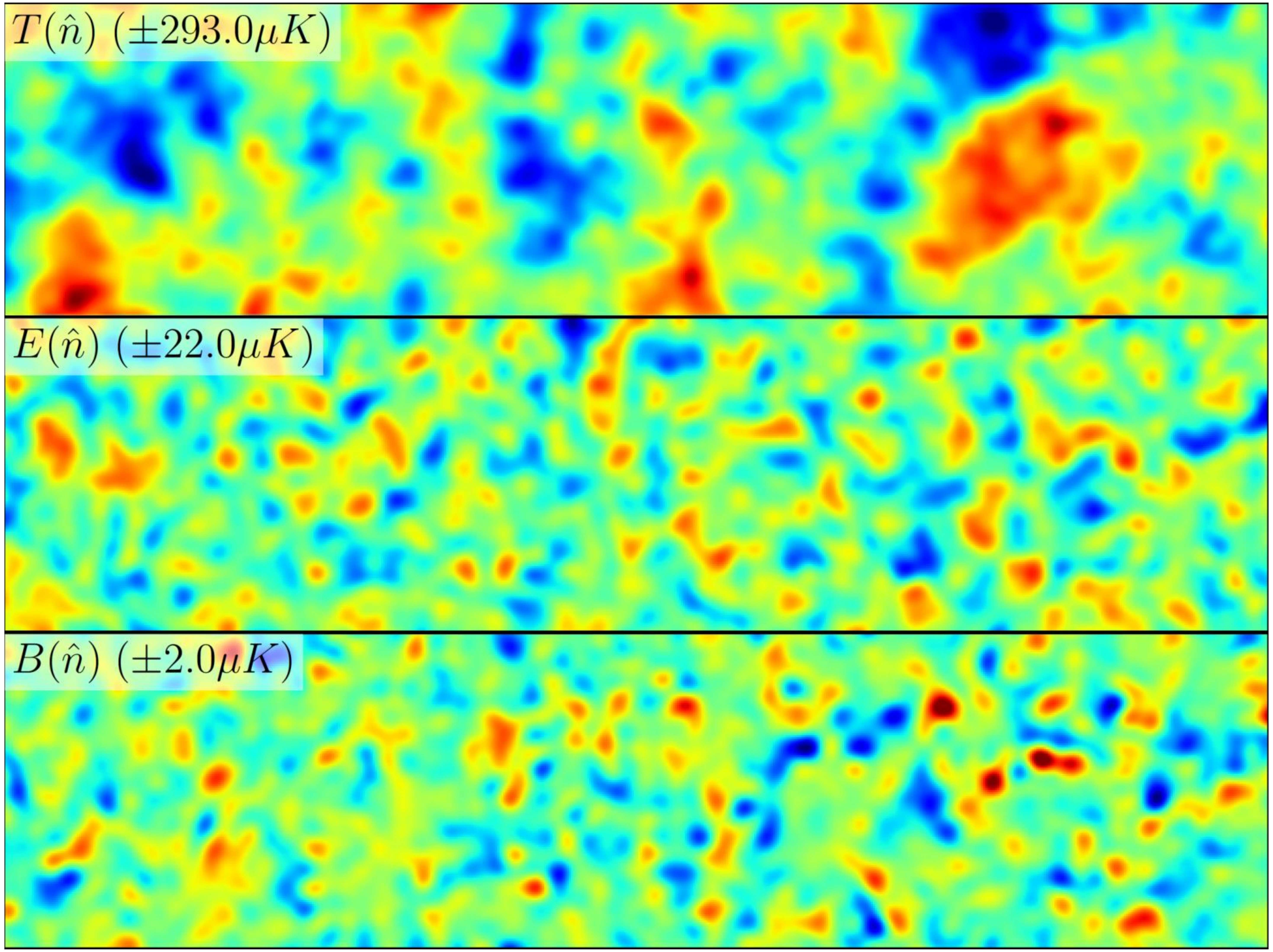
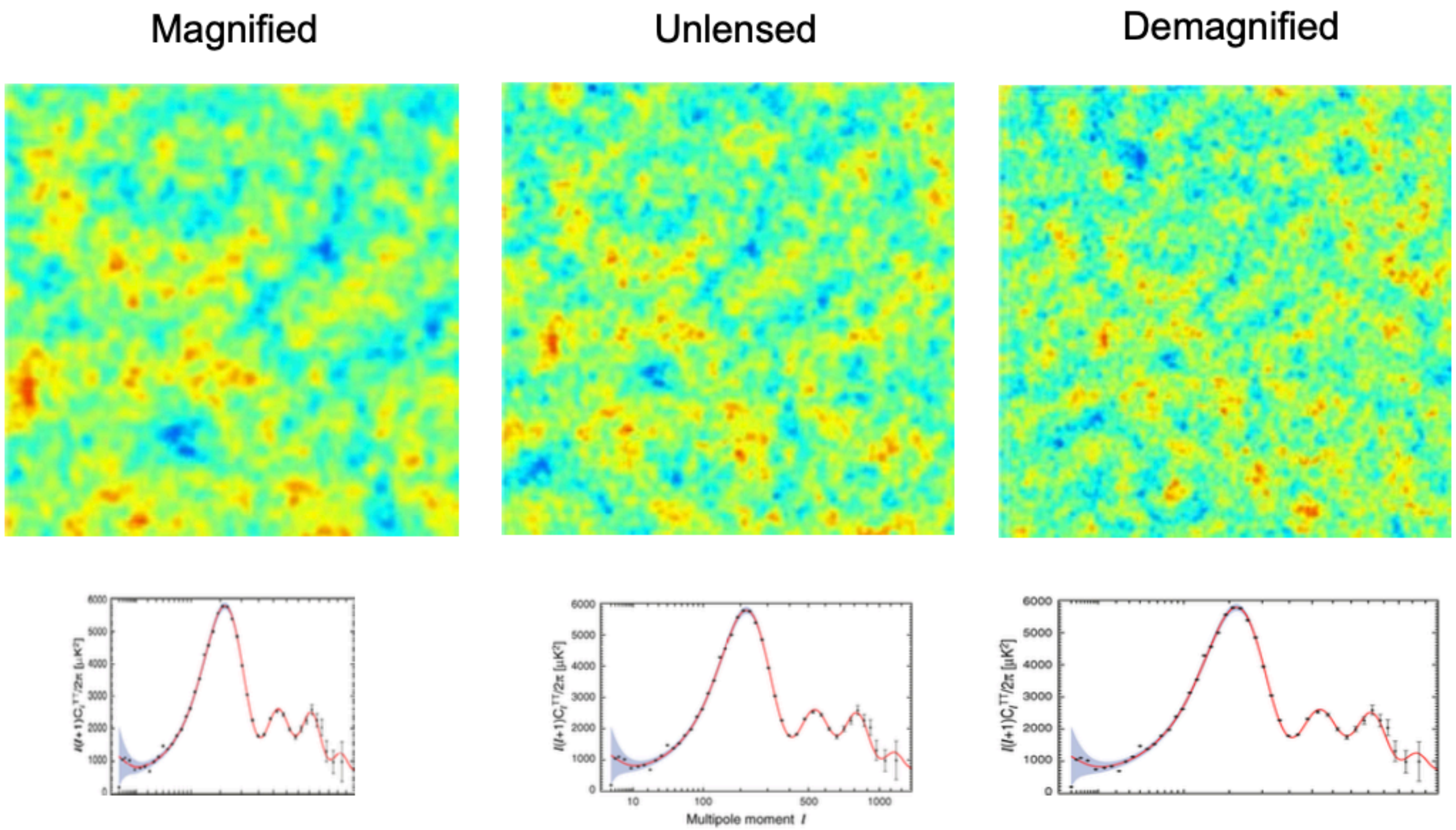


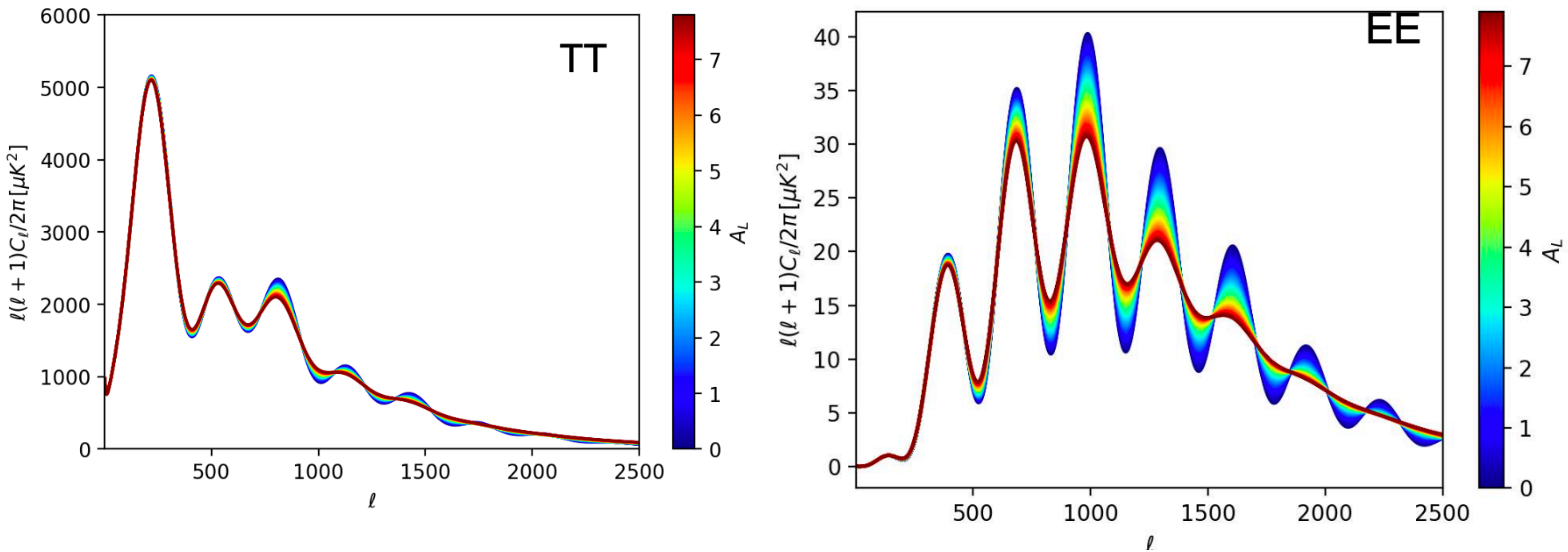
LOCAL EFFECT ON THE POWER SPECTRUM



A Lewis

SMOOTHING OF SPECTRUM

- Averaged over the sky, lensing smooths out the power spectrum

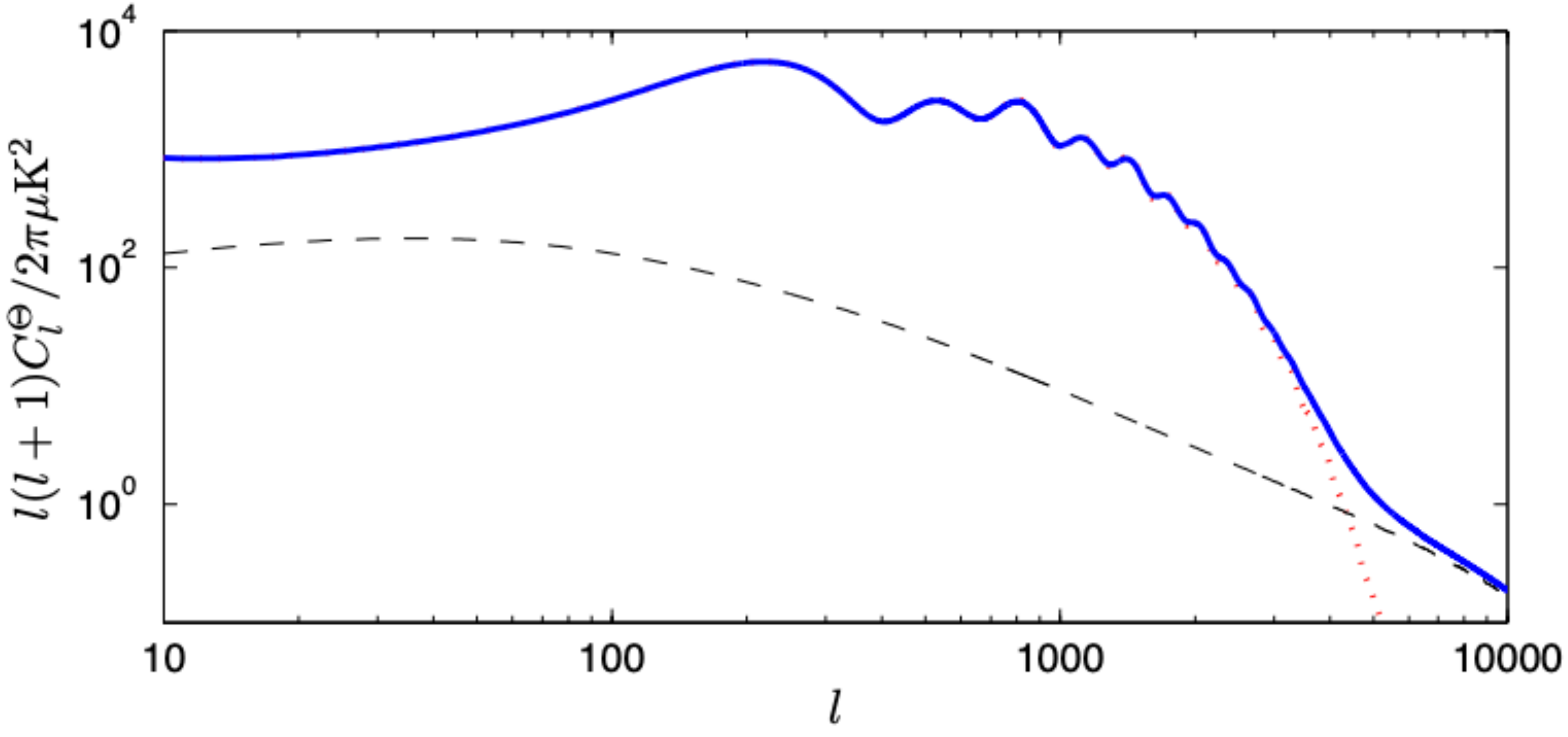


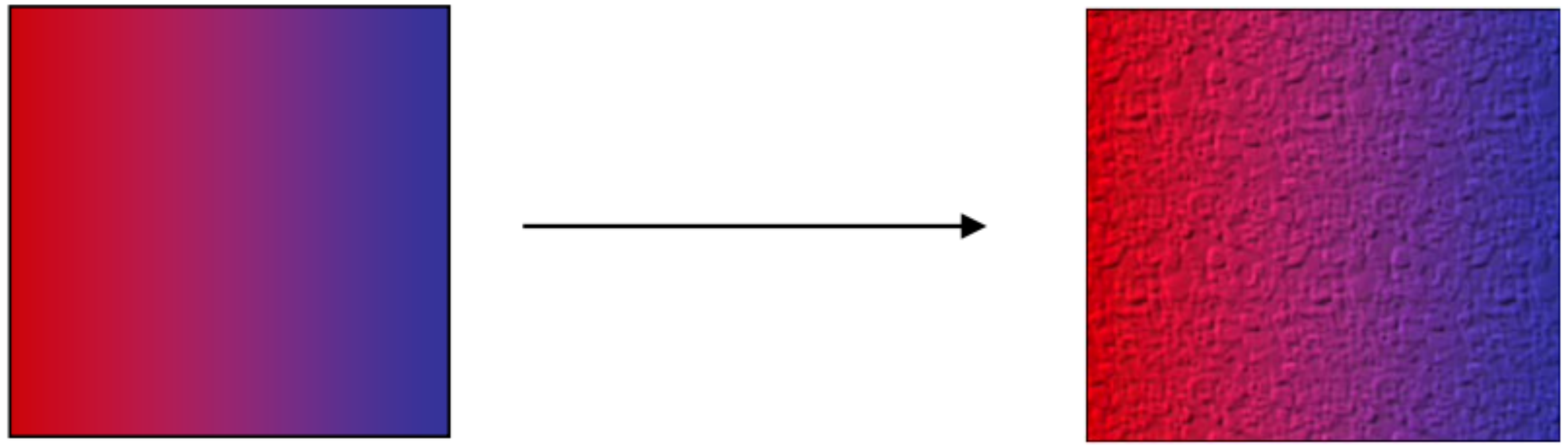
- ΛCDM prediction is $A_{\mathrm{L}} = 1$

A. Lewis

SMALL SCALES

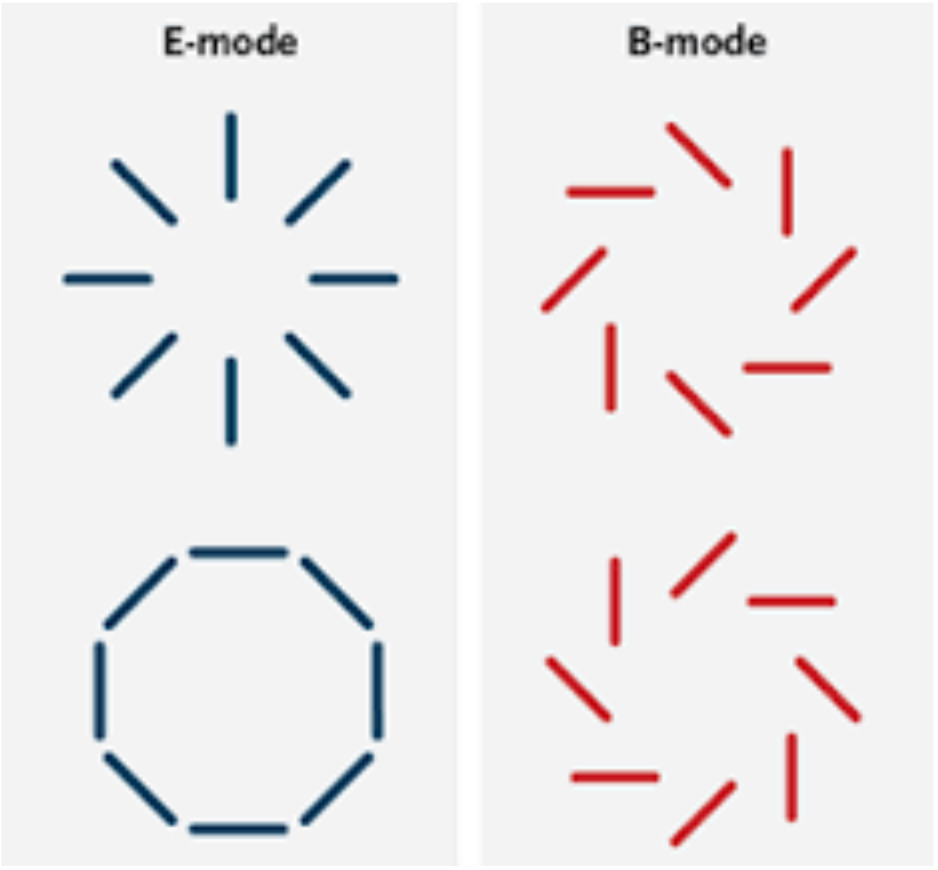
- On small scales CMB spots are large (Silk damping)
- Appears like a gradient
- CMB lensing transfer power from large scales to smaller scales

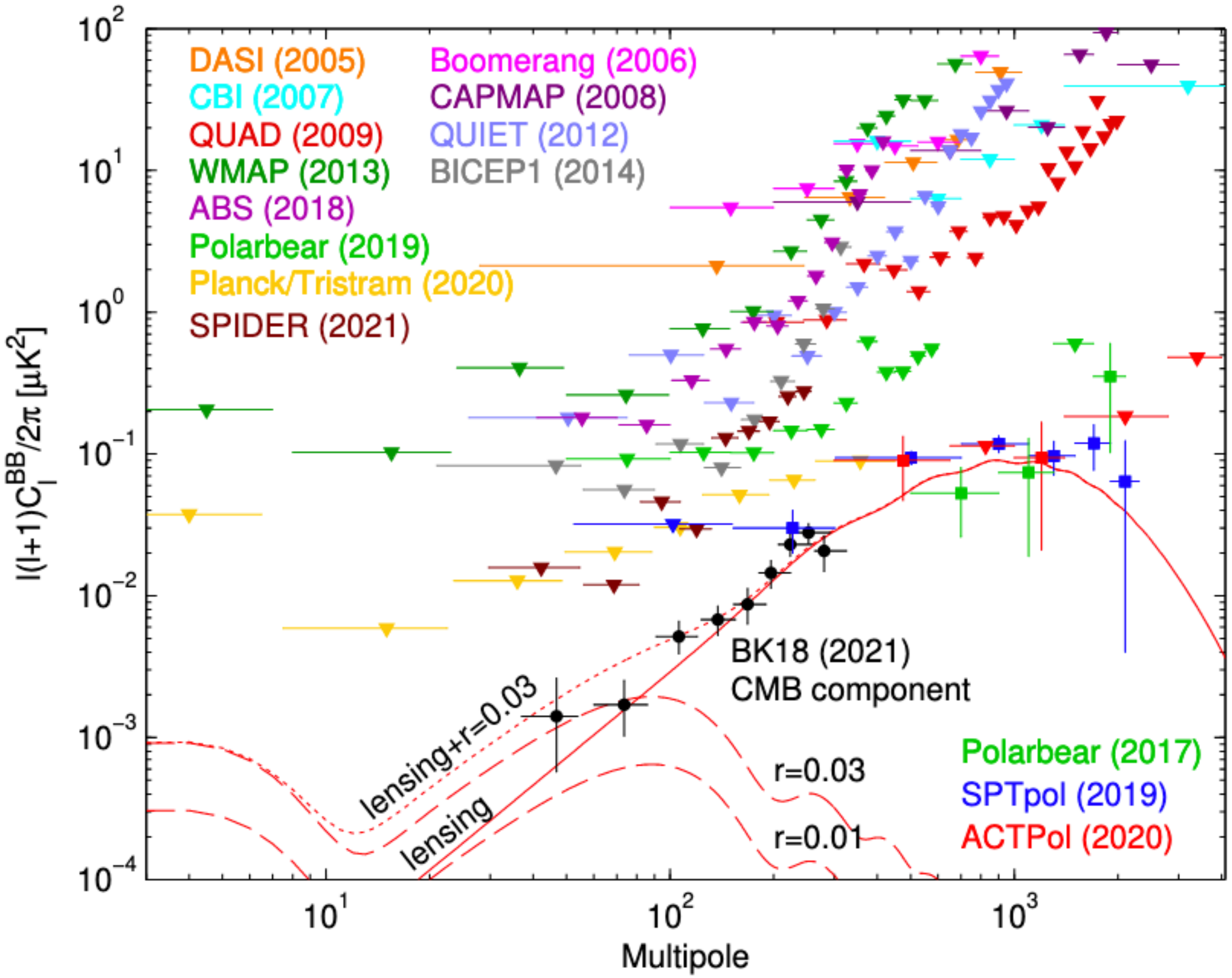




Lewis and Challinor 2006

CMB LENSING EFFECT





BICEP/Keck Collaboration 2021

- Lensing creates B modes from E modes
- Need to delens the B power spectrum to measure the primordial B modes created by the gravitational waves of inflation

LENSING POTENTIAL

- Lensing is described by the deflection field, which can be described with a potential and a curl term

$$\boldsymbol{d} = \overrightarrow{\nabla}\phi + \overrightarrow{\nabla} \times (\omega \hat{e}_z)$$

- In practice the curl term is vanishing in the Born approximation (assuming lensing happens in one plane)
- The lensing potential is the Weyl potential integrated along the line of sight

$$\phi(\boldsymbol{n}) = -2\int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi} \Psi\left(\chi \boldsymbol{n}; \eta_0 - \chi\right)$$

- We also often use the convergence field

$$\kappa \equiv -\frac{1}{2}\overrightarrow{\nabla}\cdot\boldsymbol{d} = -\frac{1}{2}\Delta\phi$$

CMB LENSING POWER SPECTRUM

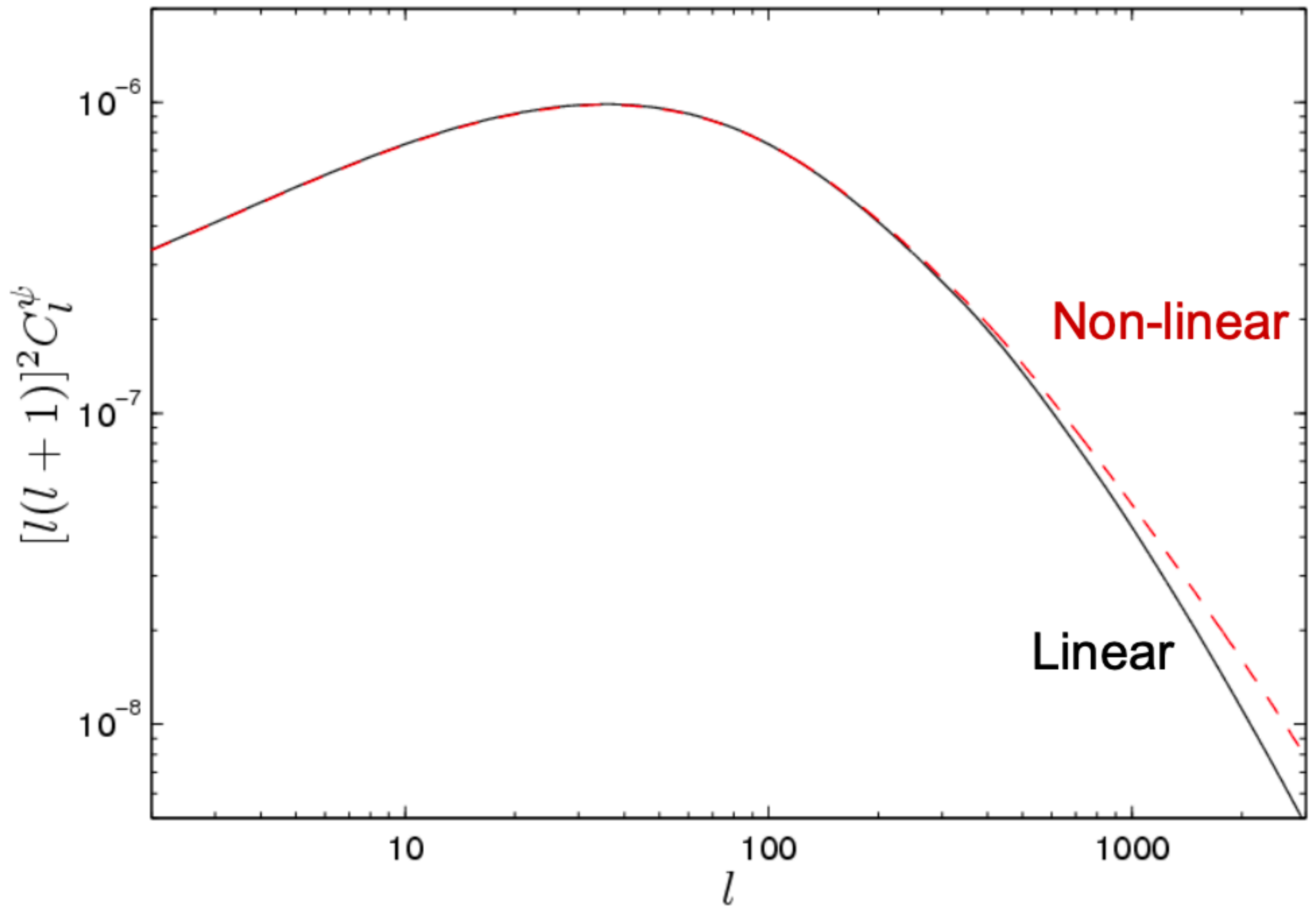
Limber approximation

$$C_l^{\psi} \approx \frac{8\pi^2}{l^3} \int_0^{\chi_*} \chi \mathrm{d}\chi \, \mathcal{P}_\Psi(l/\chi; \eta_0 - \chi) \left(\frac{\chi_* - \chi}{\chi_* \chi} \right)^2$$

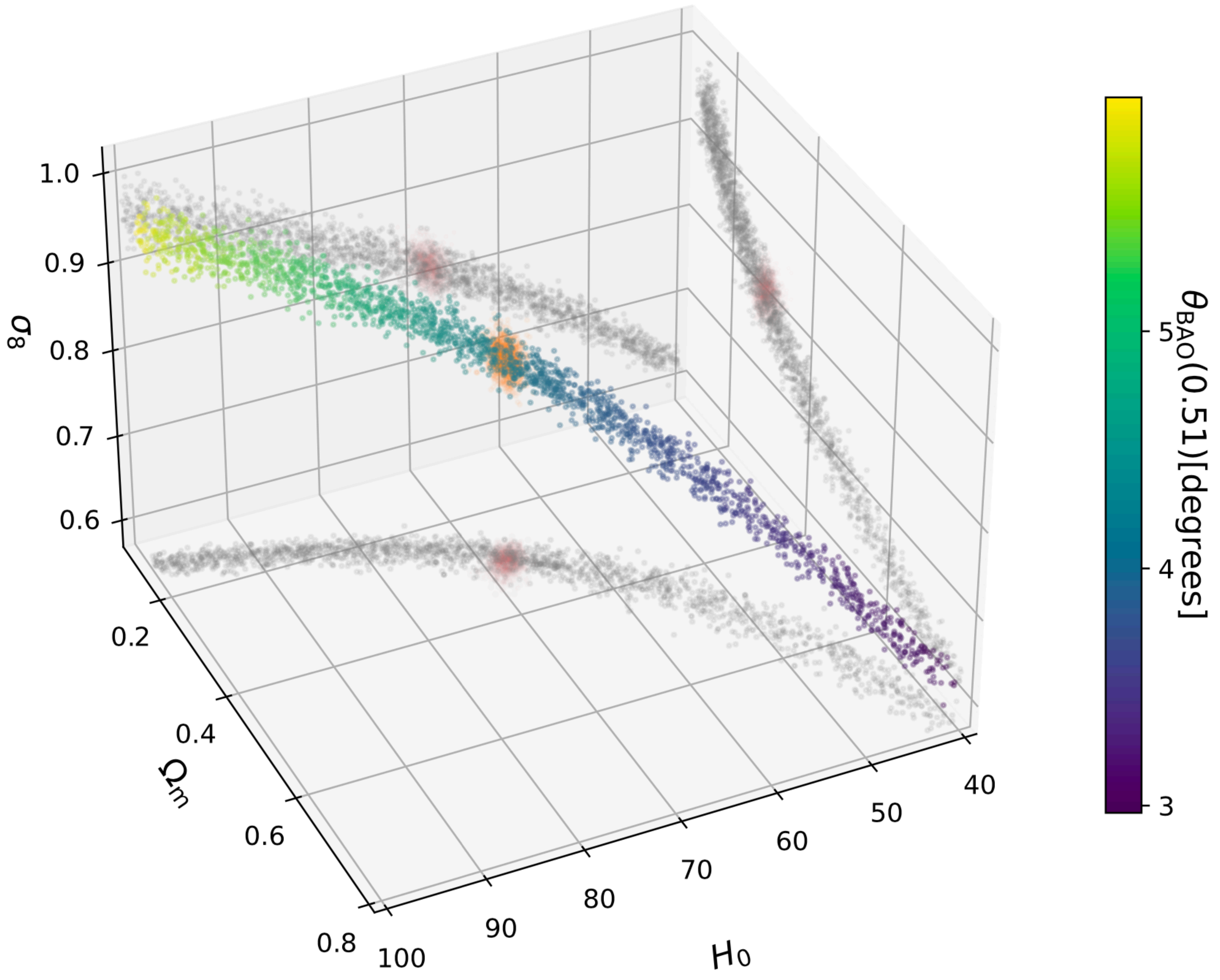
Poisson equation gives

$$\mathcal{P}_\Psi(k;\eta) = \frac{9\Omega_m^2(\eta) H^4(\eta)}{4} \frac{\mathcal{P}_{\bar{\delta}}(k;\eta)}{k^4} = \frac{9\Omega_m^2(\eta) H^4(\eta)}{8\pi^2} \frac{P(k;\eta)}{k},$$

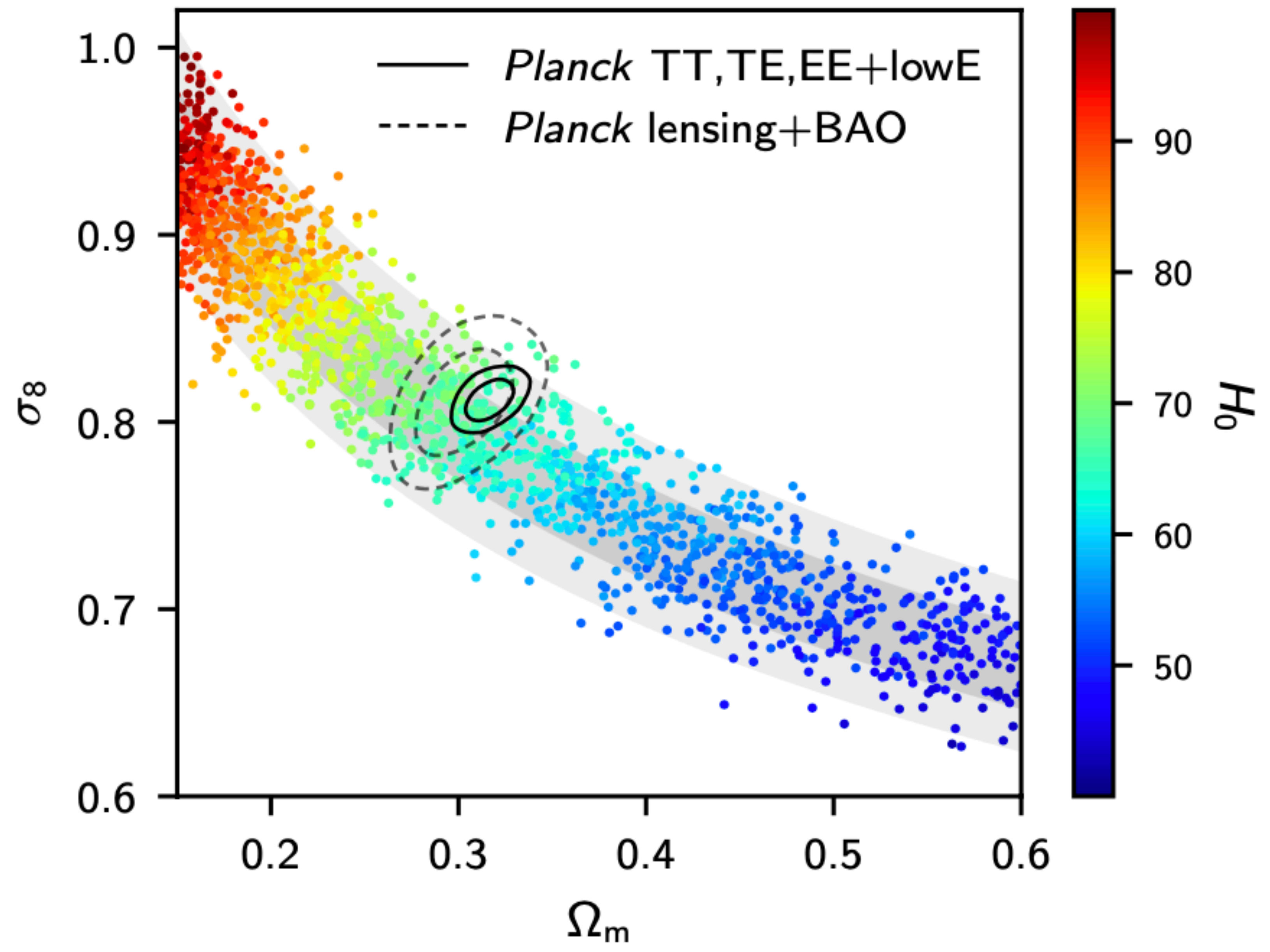
- Spectrum peaks at l = 30
  ~ coherent deflection over a few degrees
- RMS of deflection are of ~2.5 arcmin
- Kernel peaks at z~2 => LSS are mostly linear



WHAT CMB LENSING MEASURES



Carron et al 2022



Planck CMB lensing alone measures

$$\sigma_8 \Omega_m^{0.25} = 0.589 \pm 0.020$$

"Lensing-only" priors

$$\Omega_b h^2 = 0.0222 \pm 0.0005;$$

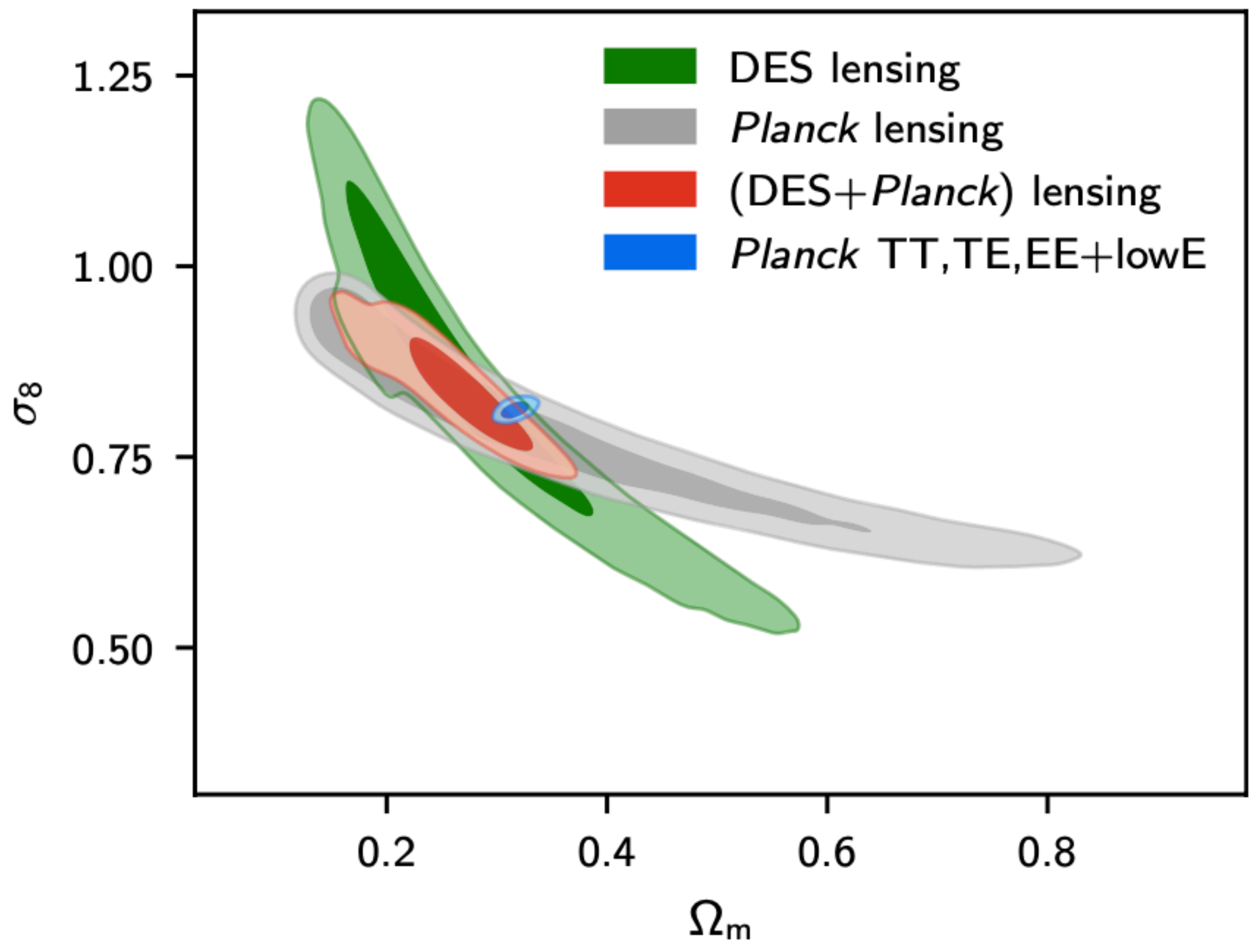
$$n_s = 0.96 \pm 0.02;$$

$$0.4 < h < 1$$

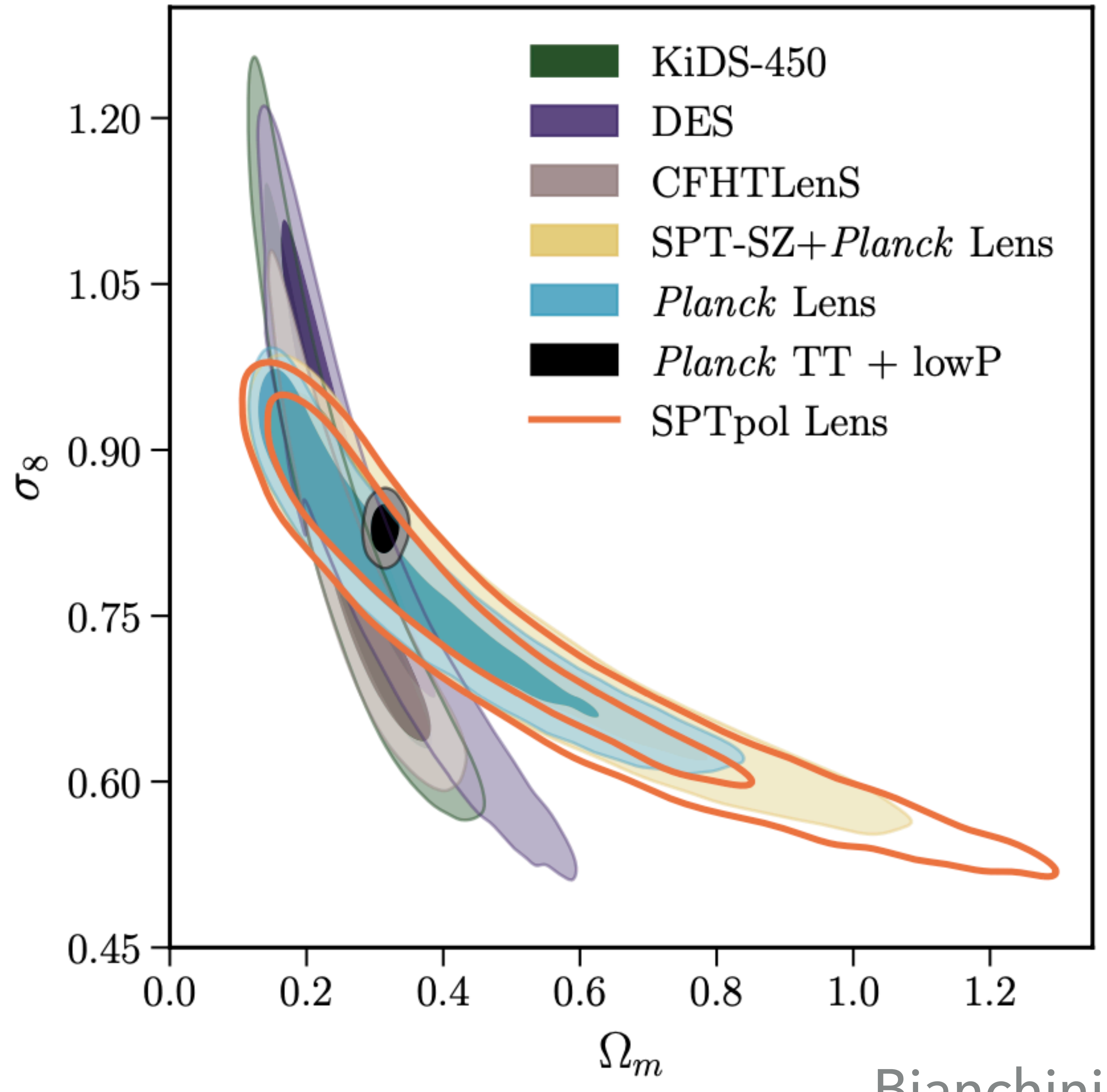
Planck collaboration 2018

LENSING CONSTRAINTS

- Degeneracy breaking with galaxy lensing surveys: same probe but at lower redshift



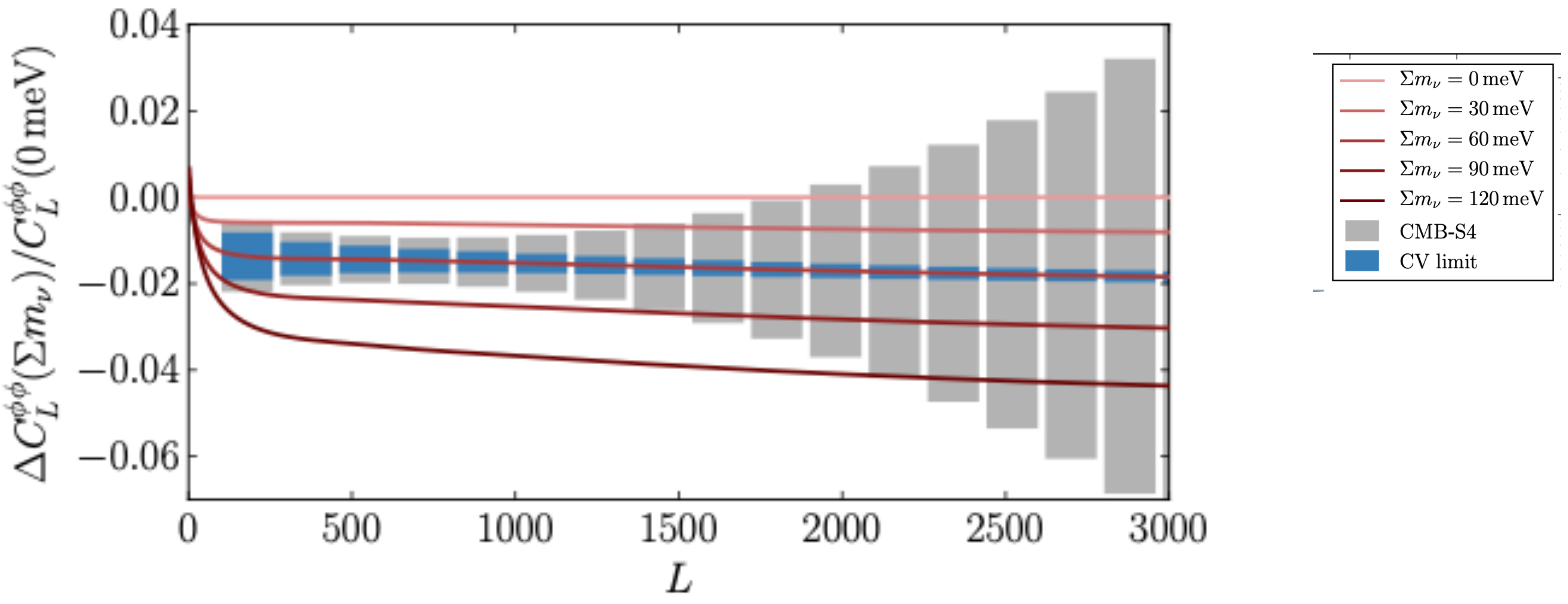
Planck collaboration 2018



Bianchini et al. 2020

THE SUM OF NEUTRINO MASSES

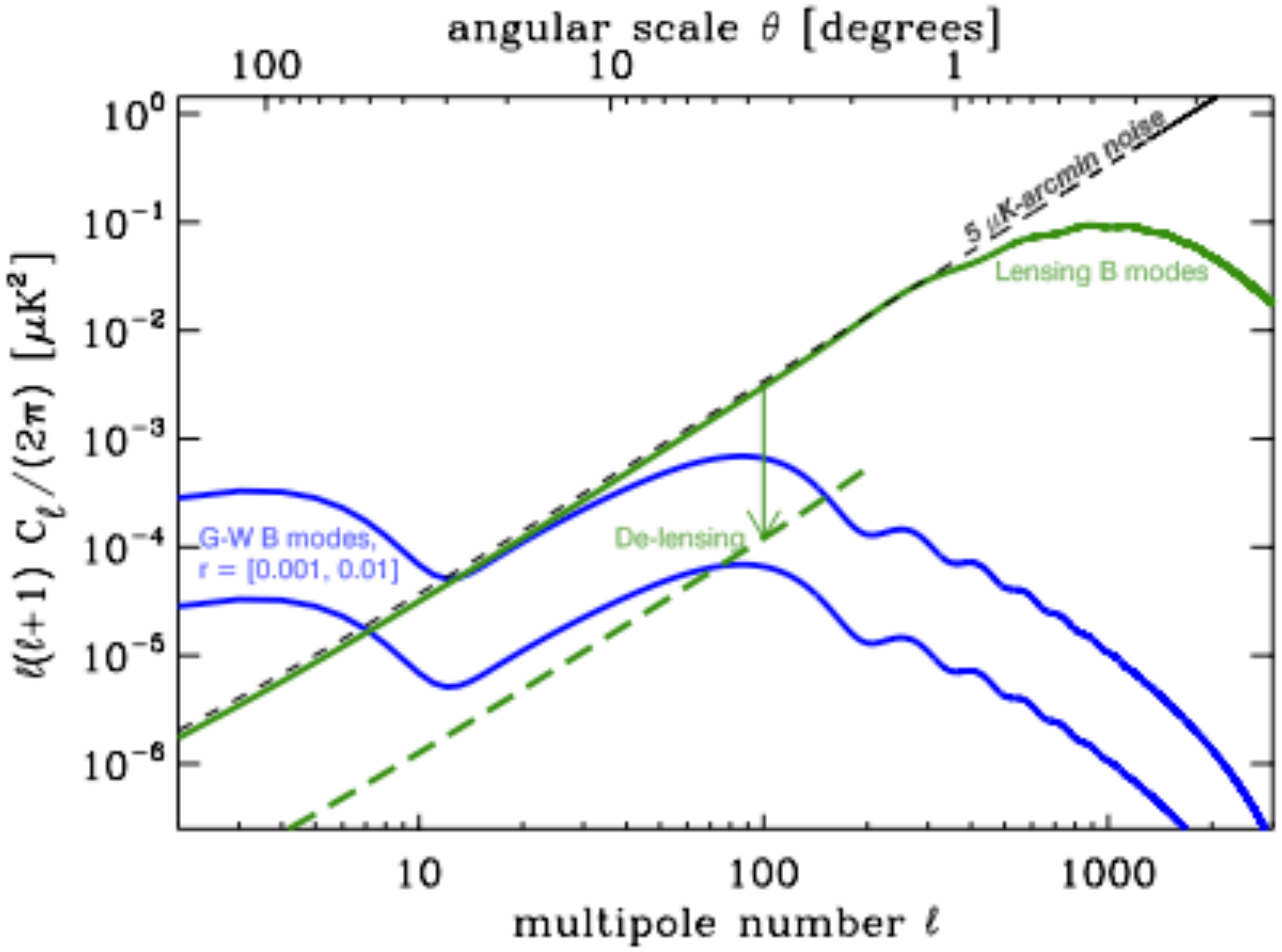
- CMB lensing is sensitive to the growth of structures, and as such to the sum of neutrino masses
- Expected 4sigma detection of massive neutrinos with CMB-S4 + CMB lensing + BAO



CMB-S4 Science book 2016

THE TENSOR TO SCALAR RATIO

- Inflation generates primordial gravitational wav
- Primordial B modes of polarisation are sourced these GW, but most of the observed B modes a coming from lensing
- Need to remove these lensing B modes (delensing)



CMB-S4 Science book 2016

OUTLINE

- What is the gravitational lensing of the CMB ?
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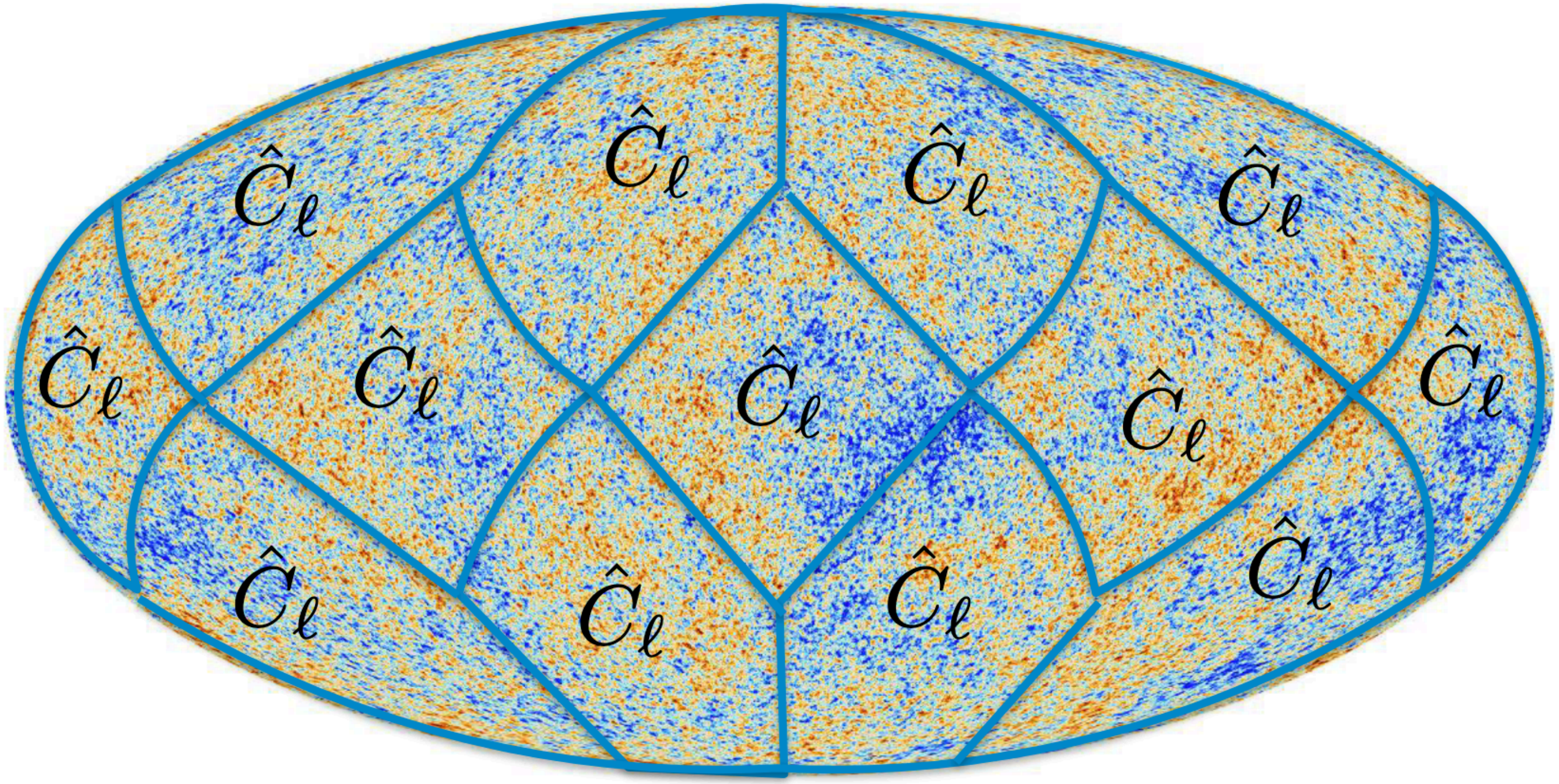
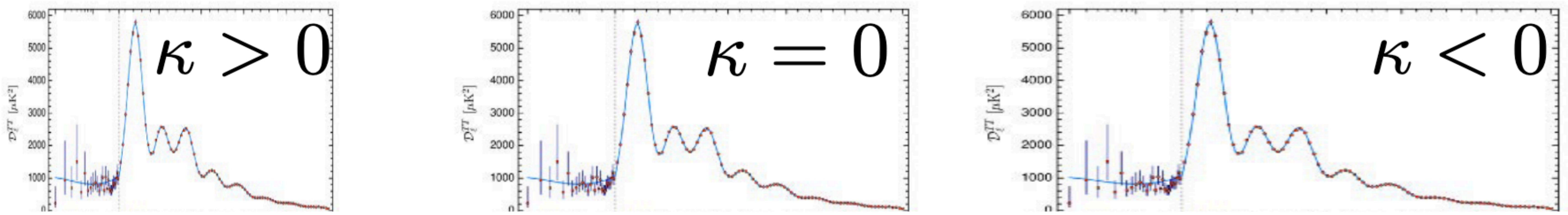
BASIC IDEA OF QUADRATIC ESTIMATION

- Lensing creates statistical anisotropies: patches of the sky will have different spectra
- We can measure these deviations, and estimate the lensing potential field





QUADRATIC ESTIMATOR (QE)

- Lensing creates correlations between different multipole moments

$$\left\langle X^{\mathrm{len}}(\boldsymbol{l})\, Y^{\mathrm{len}\,*}(\boldsymbol{l}')\right\rangle_{\substack{\text{fixed lensed} \\ \boldsymbol{l}\neq\boldsymbol{l}',\ \boldsymbol{L}=\boldsymbol{l}+\boldsymbol{l}'}} = f_{XY}(\boldsymbol{l},\boldsymbol{l}')\,\phi(\boldsymbol{L})$$

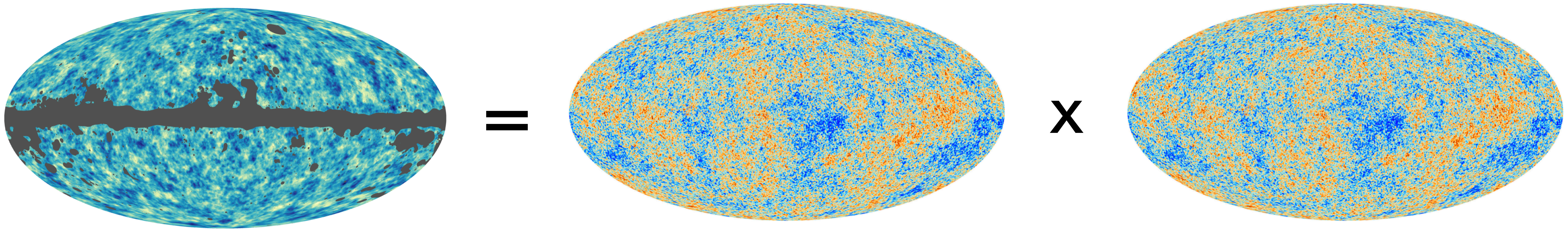
Lensing induced correlations

- The QE combines scales of two CMB fields (Hu & Okamoto 2002)

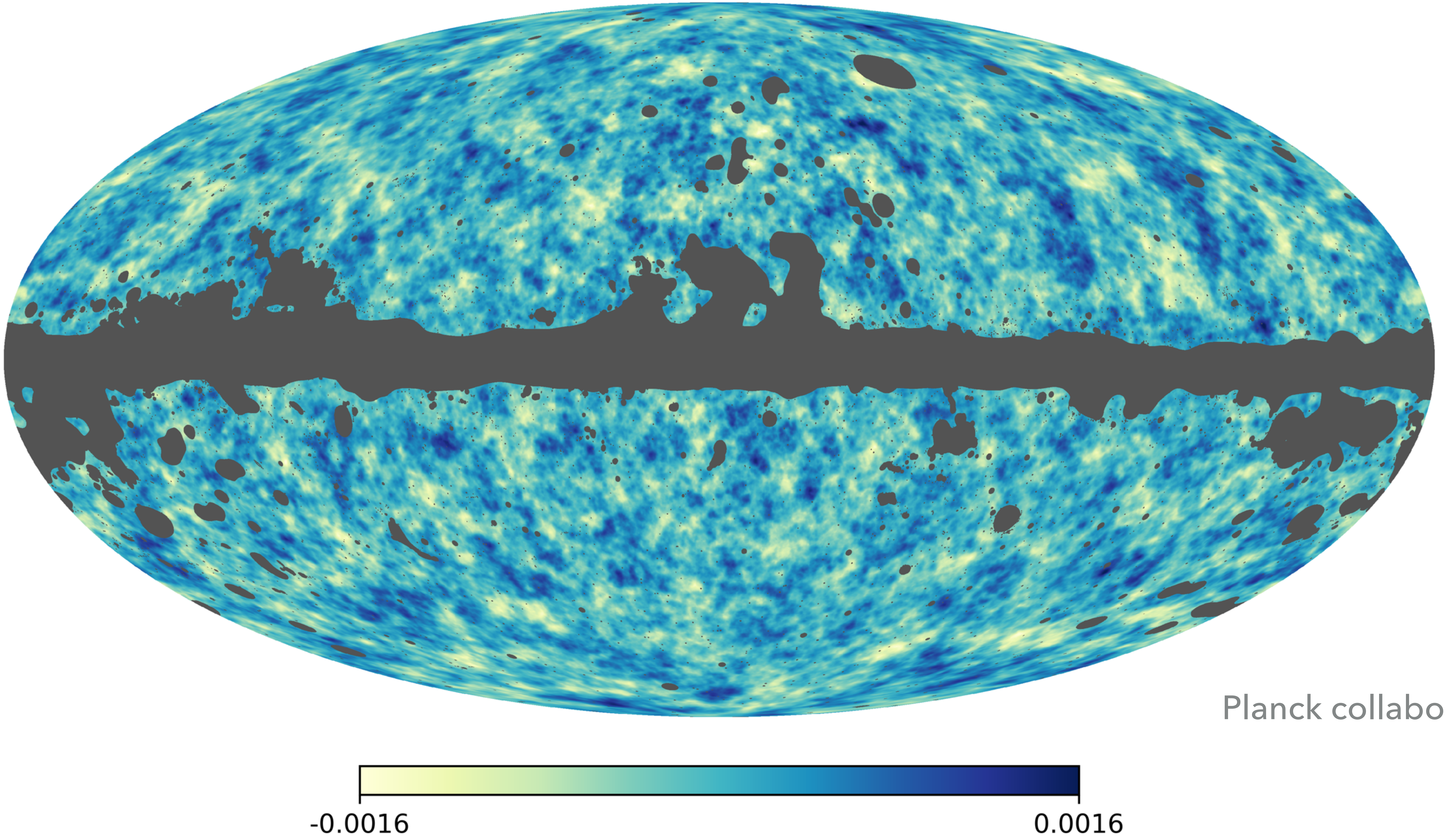
$$\hat{\phi}(\boldsymbol{L}) = \frac{1}{R_L^{XY}} \int \frac{d^2\boldsymbol{l}}{2\pi}\, f^{XY}(\boldsymbol{l},\boldsymbol{L})\,\bar{X}(\boldsymbol{l})\,\bar{Y}^*(\boldsymbol{l}-\boldsymbol{L})$$

Normalisation (response of the estimator)

Inverse variance filtered CMB fields



PLANCK LENSING MAP

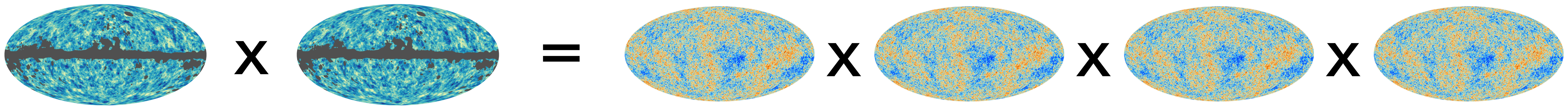


Planck collaboration 2018

NOISY RECONSTRUCTION

- The power spectrum of the estimated lensing potential is a 4 point functions of the maps

$$C_L^{\hat{\phi}\hat{\phi}} =$$



- Chance correlations between different scales can mimic the lensing effect

The signal we want

$$C_L^{\hat{\phi}\hat{\phi}} = C_L^{\phi\phi} + N_L^0 + N_L^1 + \dots$$

Disconnected (gaussian) contractions of the lensed CMB fields

Non gaussian secondary contractions created by lensing (proportional to $C^{\phi\phi}$)

$$N_L^{(0)XYIJ} = \frac{1}{\mathcal{R}_L^{XY}} \frac{1}{\mathcal{R}_L^{IJ}} \int \frac{\mathrm{d}^2 \boldsymbol{l}_1}{(2\pi)^2} F_{\ell_1}^X F_{\ell_2}^Y W^{XY}(\boldsymbol{l}_1, \boldsymbol{l}_2) \times \Big(F_{\ell_1}^I F_{\ell_2}^J W^{IJ}(\boldsymbol{l}_1, \boldsymbol{l}_2) C_{\ell_1}^{XI} C_{\ell_2}^{YJ} + F_{\ell_2}^I F_{\ell_1}^J W^{IJ}(\boldsymbol{l}_2, \boldsymbol{l}_1) C_{\ell_1}^{XJ} C_{\ell_2}^{YI} \Big) ,$$

POWER SPECTRUM BIASES

$$\hat{\phi}(\boldsymbol{L}) = \frac{1}{R_L^{XY}} \int \frac{d^2 \boldsymbol{l}}{2\pi} F^{XY}(\boldsymbol{l}, \boldsymbol{L} - \boldsymbol{l})\, X(\boldsymbol{l})\, Y^*(\boldsymbol{l} - \boldsymbol{L})$$

$$< \hat{\phi}^{XY}(L_1), \hat{\phi}^{CD}(L_2) > = \frac{1}{R_{L_1}^{XY} R_{L_2}^{CD}} \int_{l_1, l_2} \frac{d^2 l_1}{(2\pi)^2} \frac{d^2 l_2}{(2\pi)^2} F^{XY}(l_1, L_1 - l_1) F^{CD}(l_2, L_2 - l_2) < X(l_1) Y(L_1 - l_1) C(l_2) D(L_2 - l_2) >$$

$$< \hat{\phi}^{XY}, \hat{\phi}^{CD} > \simeq < XYCD >$$

N0 bias = **all disconnect contractions** of the 4 point function.
Considers the lensed CMB fields as independent Gaussian fields. We can use Wick's theorem:

$$< \hat{\phi}^{XY}, \hat{\phi}^{CD} > \simeq < XYCD > = < XY > < CD > + < XC > < YD > + < XD > < YC >$$

$$N_L^{(0)XYCD} = = \frac{1}{R_{L_1}^{XY} R_{L_2}^{CD}} \int_{l_1} \frac{d^2 l_2}{(2\pi)^2} F^{XY}(l_1, L_1 - l_1) \times \left[F^{CD}(-l_1, l_1 - L_1) C_{l_1}^{XC} C_{L_1 - l_1}^{YD} + F^{CD}(l_1 - L_1, -l_1) C_{l_1}^{XD} C_{L_1 - l_1}^{YC} \right]$$

POWER SPECTRUM BIASES

Higher order biases = connected contractions.

The N1 bias is the dominant one, and can be obtained with all contractions at order 1 in $C_L^{\phi\phi}$

$$< \hat{\phi}^{XY}, \hat{\phi}^{CD} > \simeq < XYCD > \qquad X^{\rm len} \sim X^{\rm unl} + \nabla\phi\nabla X$$

$$< \hat{\phi}^{XY}, \hat{\phi}^{CD} > \simeq < (X + \nabla\phi\nabla X)(Y + \nabla\phi\nabla Y)(C + \nabla\phi\nabla D)(D + \nabla\phi\nabla D) >$$

x4

+ all the 4 connected pairs of ϕ will give $C_L^{\phi\phi}$

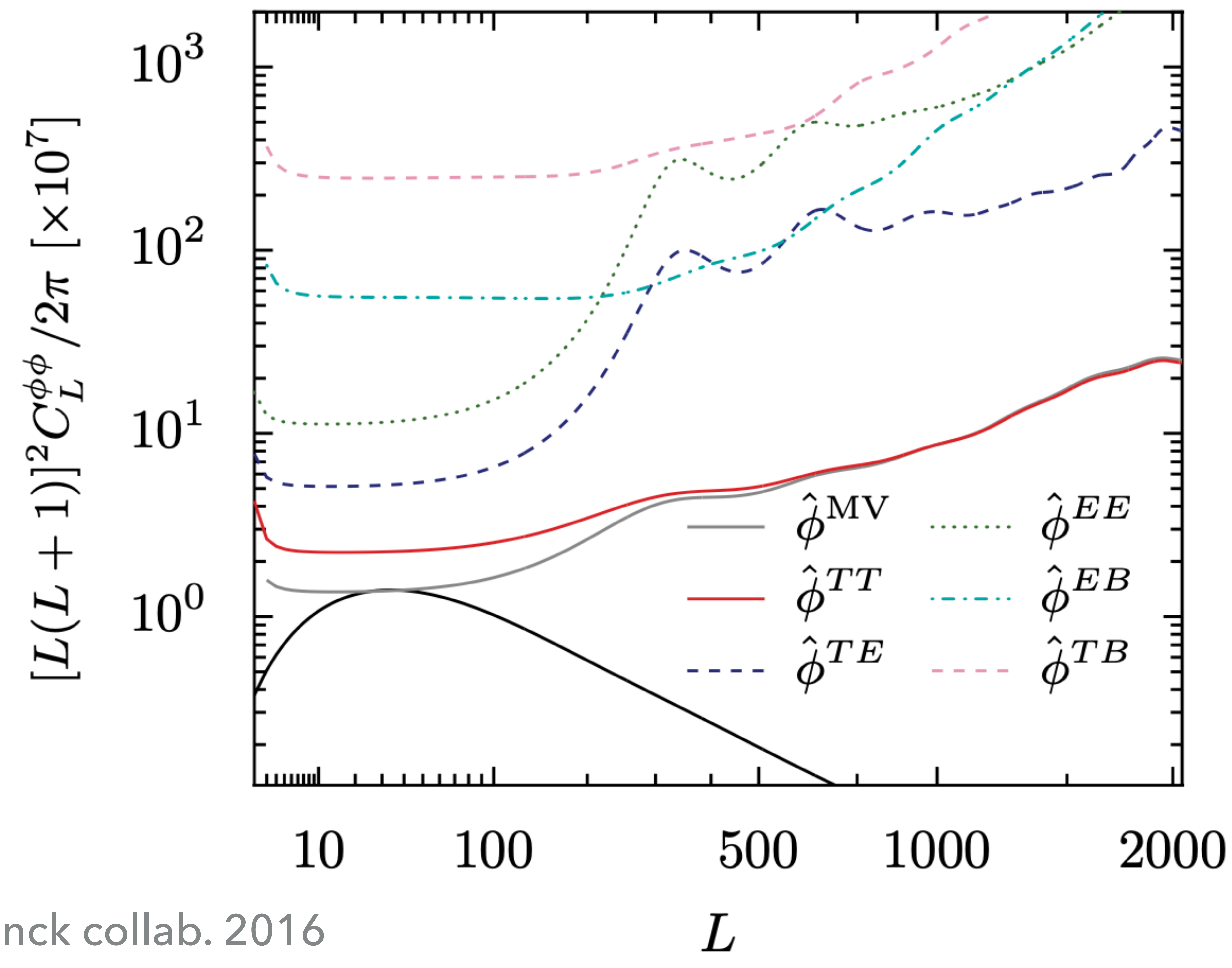
x2

$$< \hat{\phi}^{XY}, \hat{\phi}^{CD} > \simeq < (X + \nabla\phi\nabla X)(Y + \nabla\phi\nabla Y)(C + \nabla\phi\nabla D)(D + \nabla\phi\nabla D) >$$

x4

+ the combination of XD, YC and the 4 connected pairs of ϕ will give $N_L^{1,XYCD}$

NOISY RECONSTRUCTION

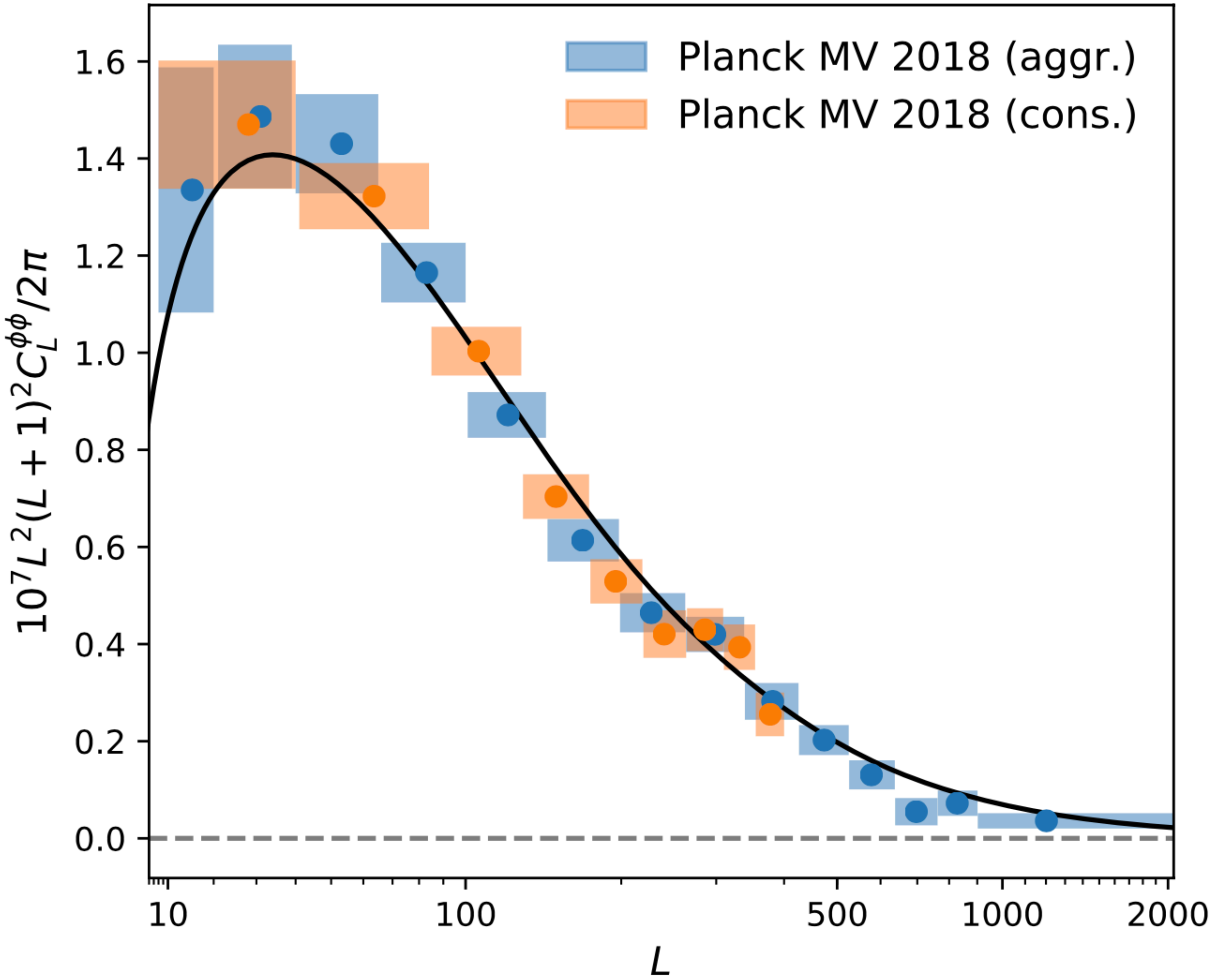


Planck collab. 2016

- Planck lensing power spectrum is dominated by the N0 bias at all scales
- Combining all pairs of maps into a minimum variance estimator
- TT estimator is dominating in Planck

PLANCK LENSING SPECTRUM

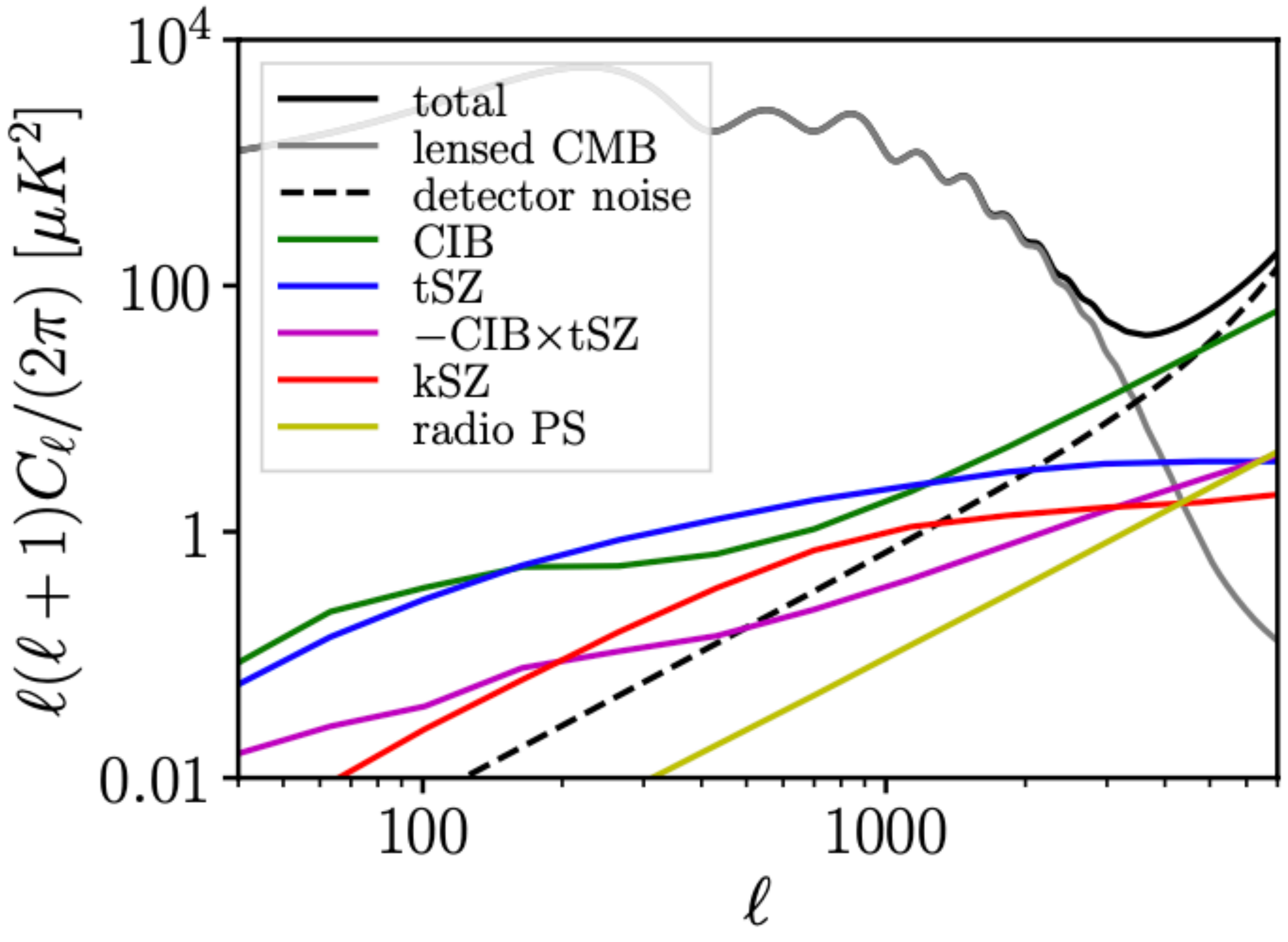
- After subtracting N0 bias
- 40 sigma detection of lensing power spectrum
- Black line here is a LCDM fit of TT, TE and EE spectra



Planck collab. 2018

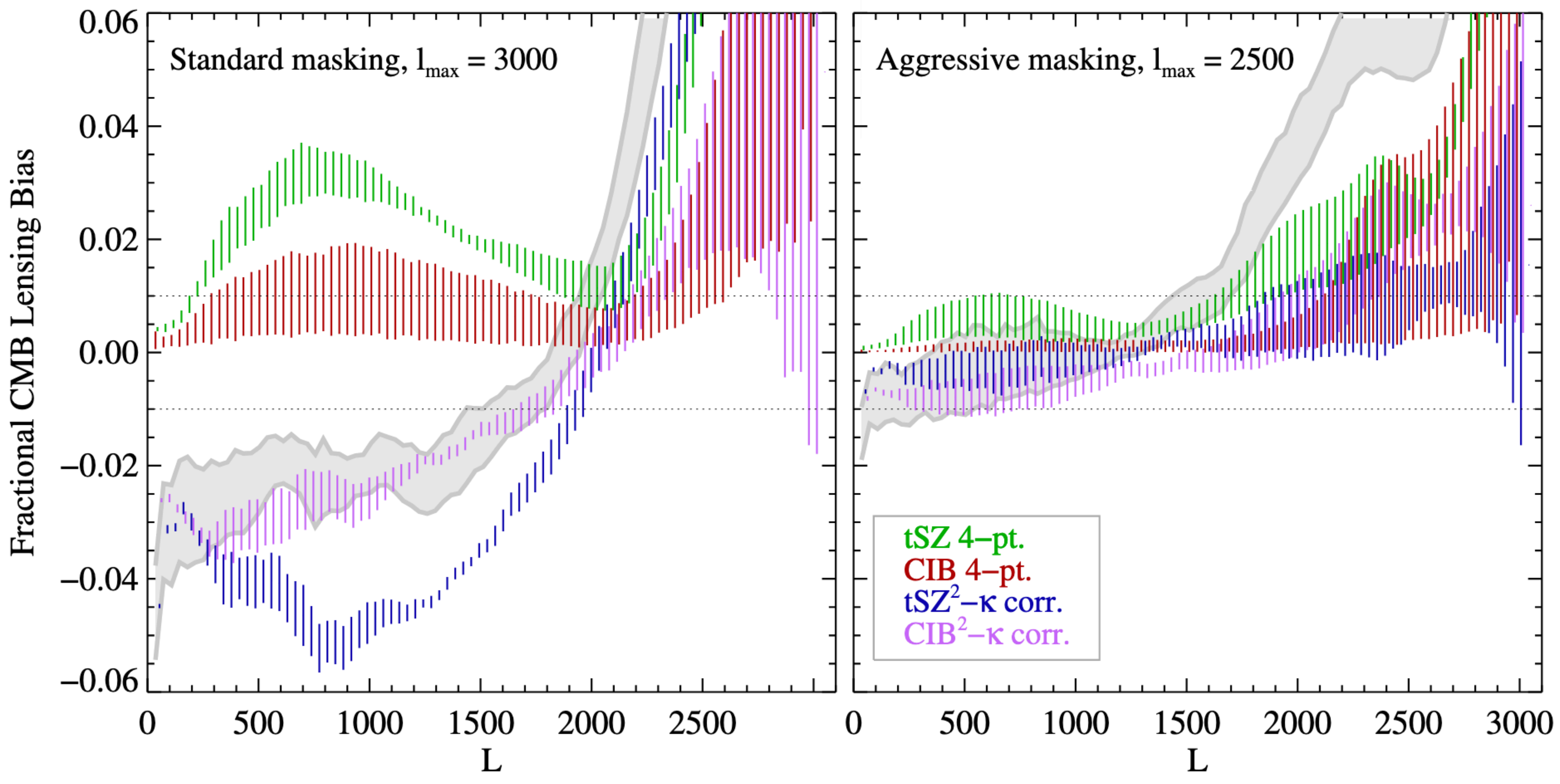
FOREGROUNDS CONTAMINATIONS

- Non gaussian distribution of foregrounds bias the quadratic estimator reconstruction
- Need to clean the maps (at the expense of higher noise)
- For reconstructing CMB lensing we often do not use scales above l=3000 in temperature to reduce biases
- That’s also why polarisation is interesting: polarised foreground are mostly large scales so we expect that we can use up to l=5000



Schaan et al 2019

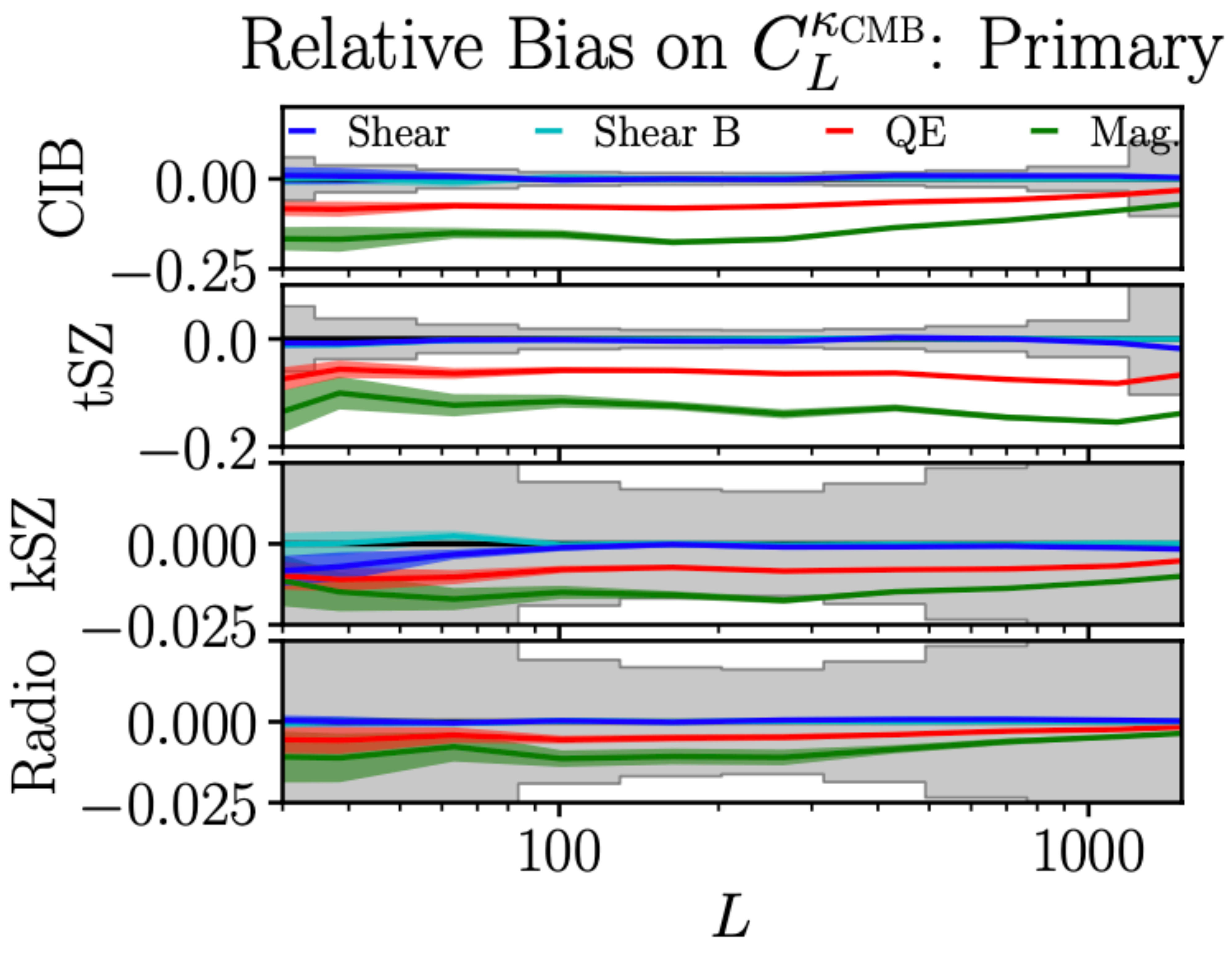
FOREGROUND BIASES



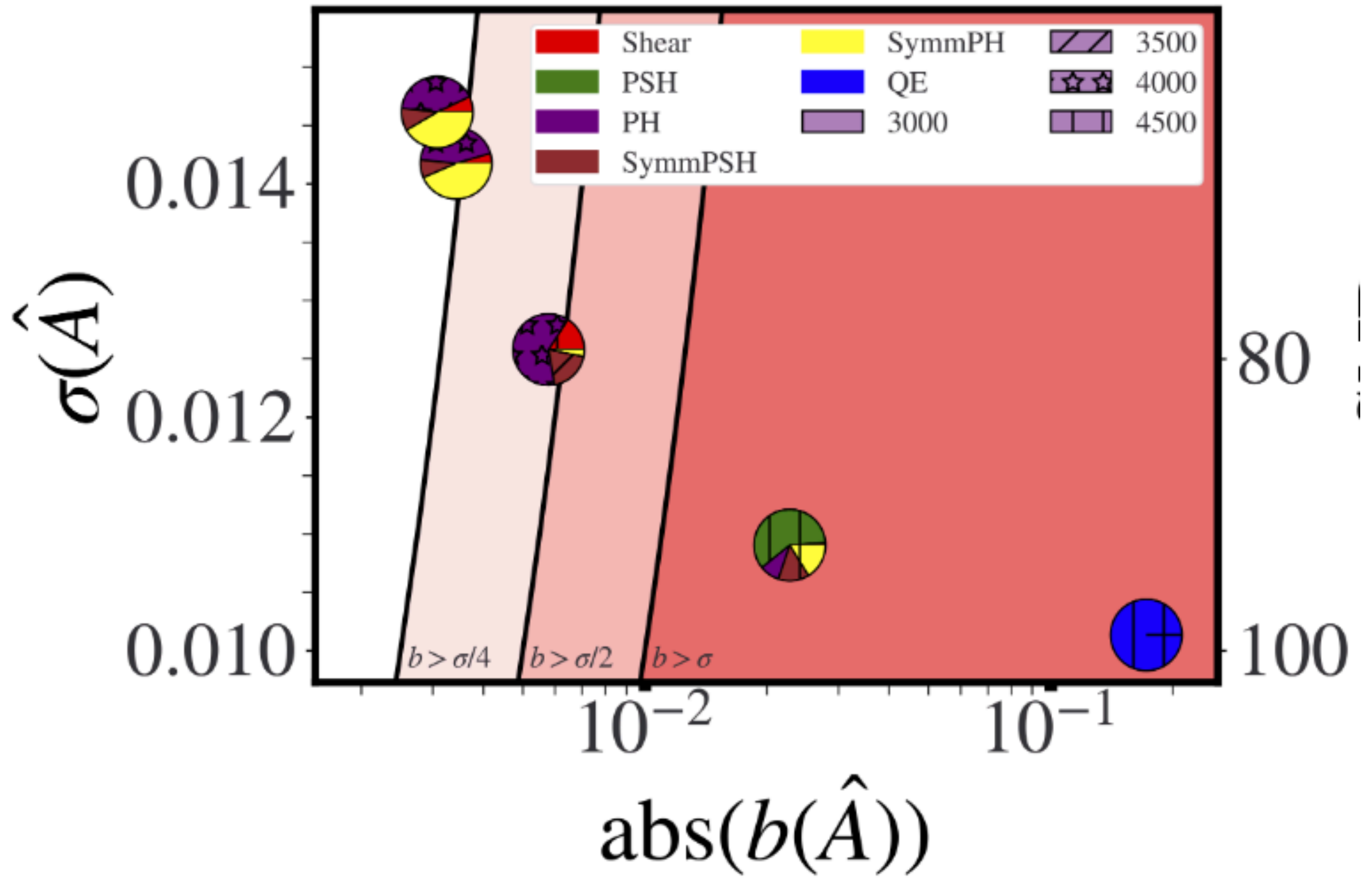
Van Engelen et al 2013

ROBUST ESTIMATORS

- Tradoff between bias and uncertainty



Schaan et al 2019



Darwish et al 2021

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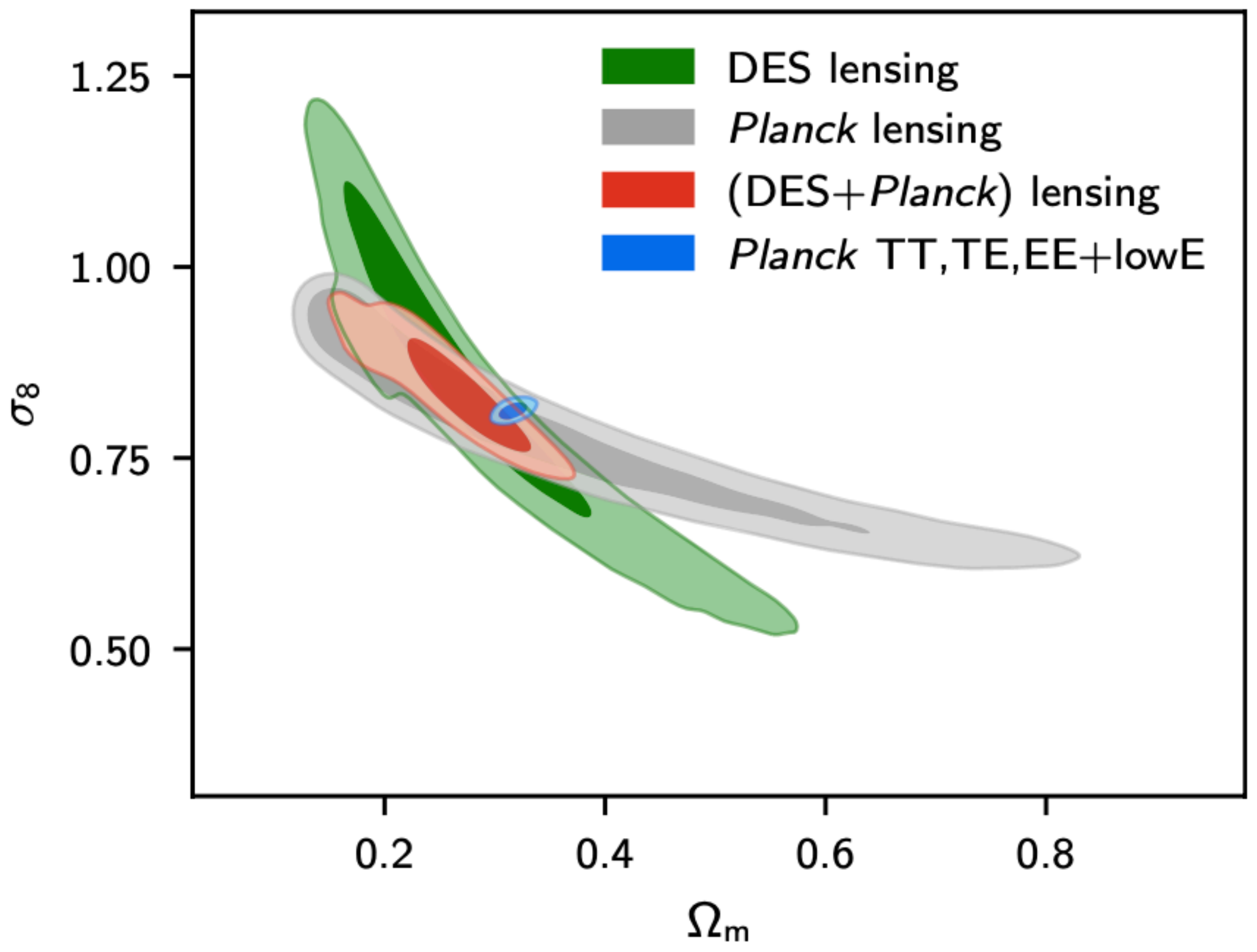
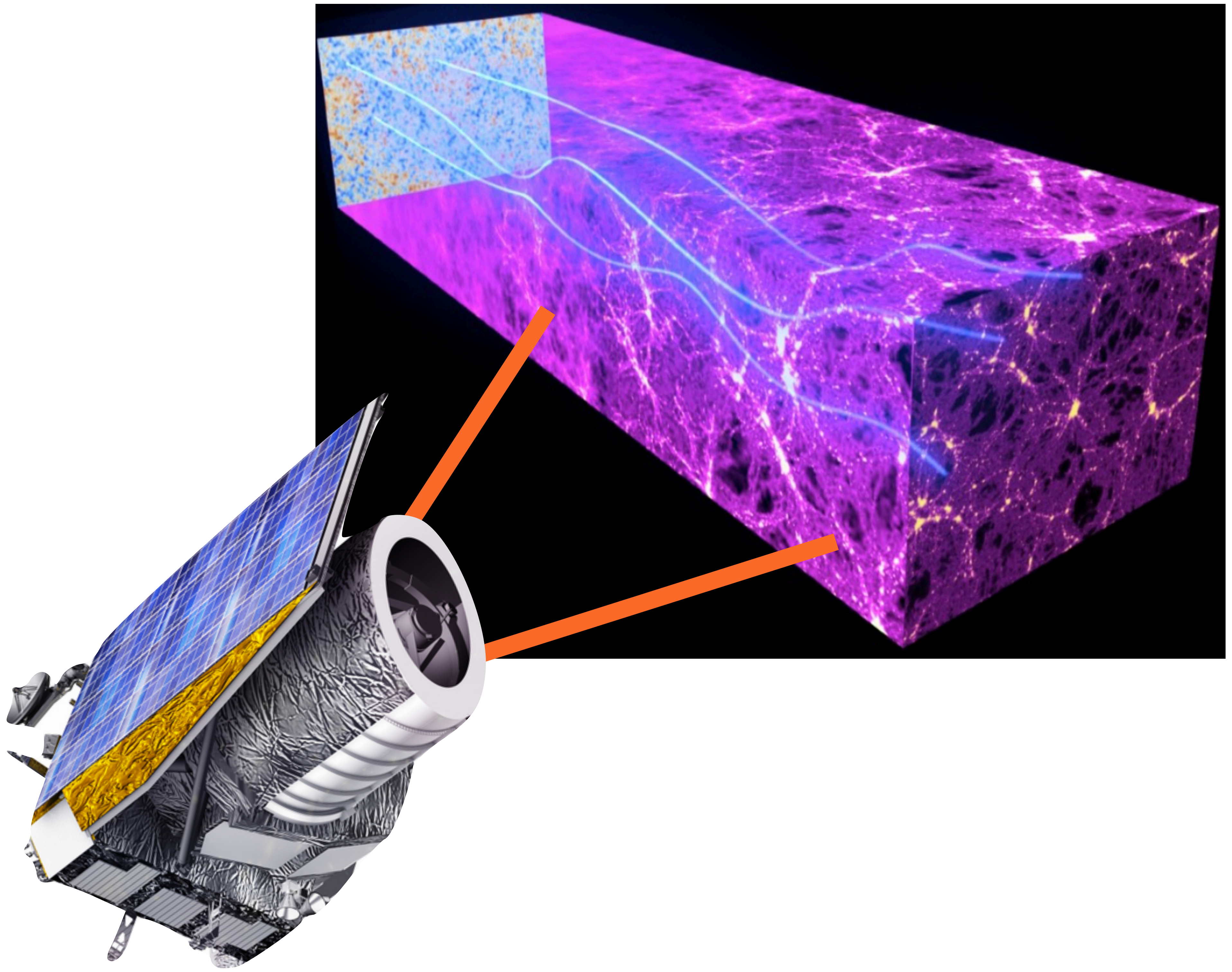
WHAT IS THE INTEREST ?

- They probe the same matter distribution
- They have different systematics
- They have different cosmological degeneracies





CROSS POWER SPECTRA

Limber approximation:

$$C_\ell^{XY} = c \int \frac{\mathrm{d}z}{H(z) r^2(z)} \mathcal{W}^X(z)\, \mathcal{W}^Y(z)\, P_{\delta\delta}\left(\frac{\ell + 1/2}{r(z)}, z\right)$$

Galaxy clustering kernel:

$$\mathcal{W}^{\mathrm{GCp}_i}(z) = b_i(z) \frac{n_i(z)}{\bar{n}_i} \frac{H(z)}{c}$$

Galaxy weak lensing kernel:

$$\mathcal{W}^{\mathrm{WL}_i}(z) = \mathcal{W}^{\gamma_i}(z) - \frac{\mathcal{P}_{\mathrm{IA}} \Omega_{\mathrm{m},0}}{D(z)} \mathcal{W}^{\mathrm{IA}_i}(z),$$

with

$$\mathcal{W}^{\gamma_i}(z) = \frac{3}{2} \frac{H_0^2}{c^2} \Omega_{\mathrm{m},0} (1+z)\, r(z) \int_z^\infty \mathrm{d}z' \frac{n_i(z')}{\bar{n}_i} \left[1 - \frac{r(z)}{r(z')}\right],$$

$$\mathcal{W}^{\mathrm{IA}_i}(z) = \frac{n_i(z)}{\bar{n}_i} \frac{H(z)}{c}.$$

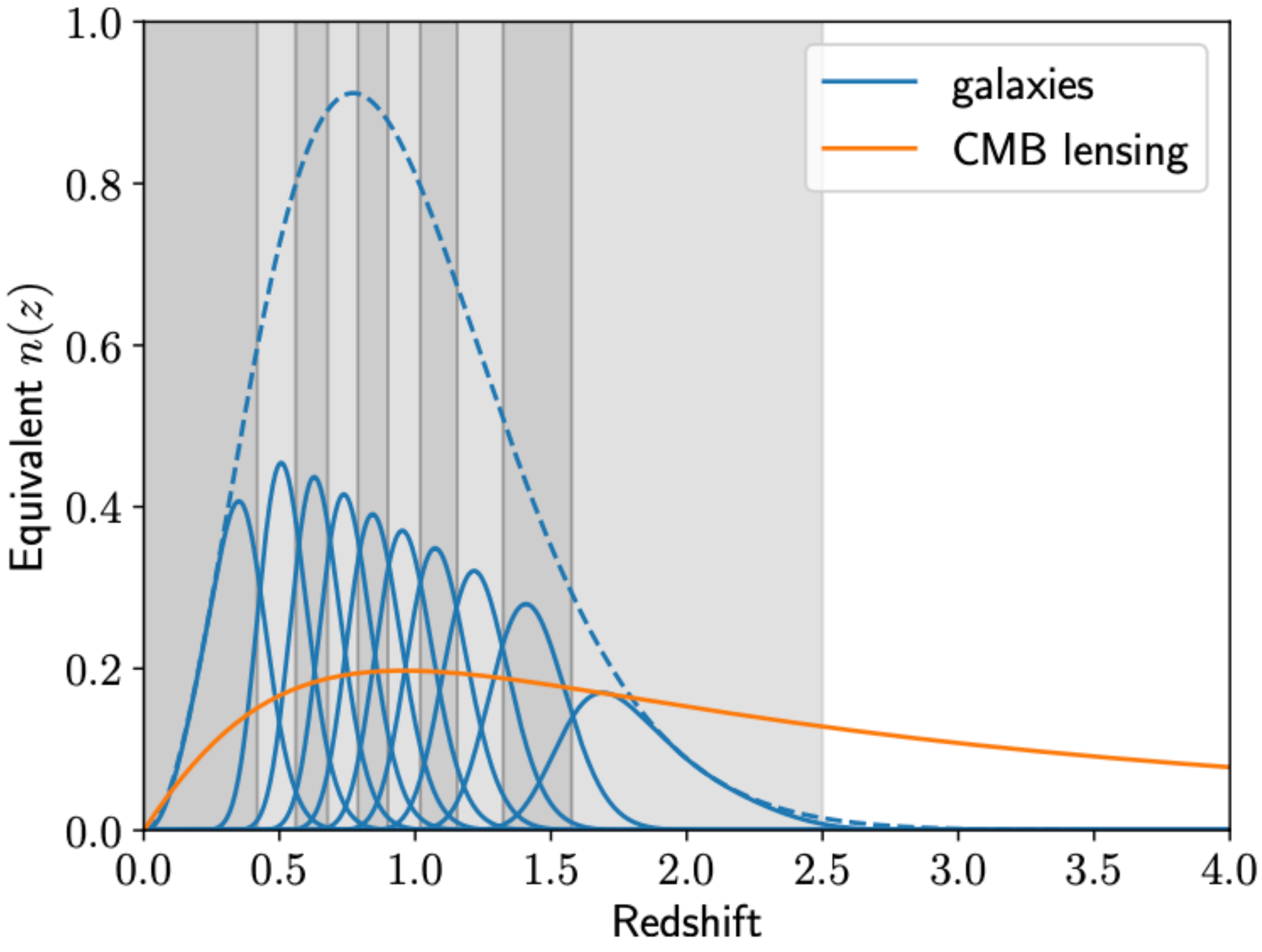
CMB lensing kernel:

$$\mathcal{W}^{\phi}(z) = \frac{3}{2} \frac{H_0^2}{c^2} \Omega_{\mathrm{m},0} (1+z)\, r(z) \left[1 - \frac{r(z)}{r(z^*)}\right]$$

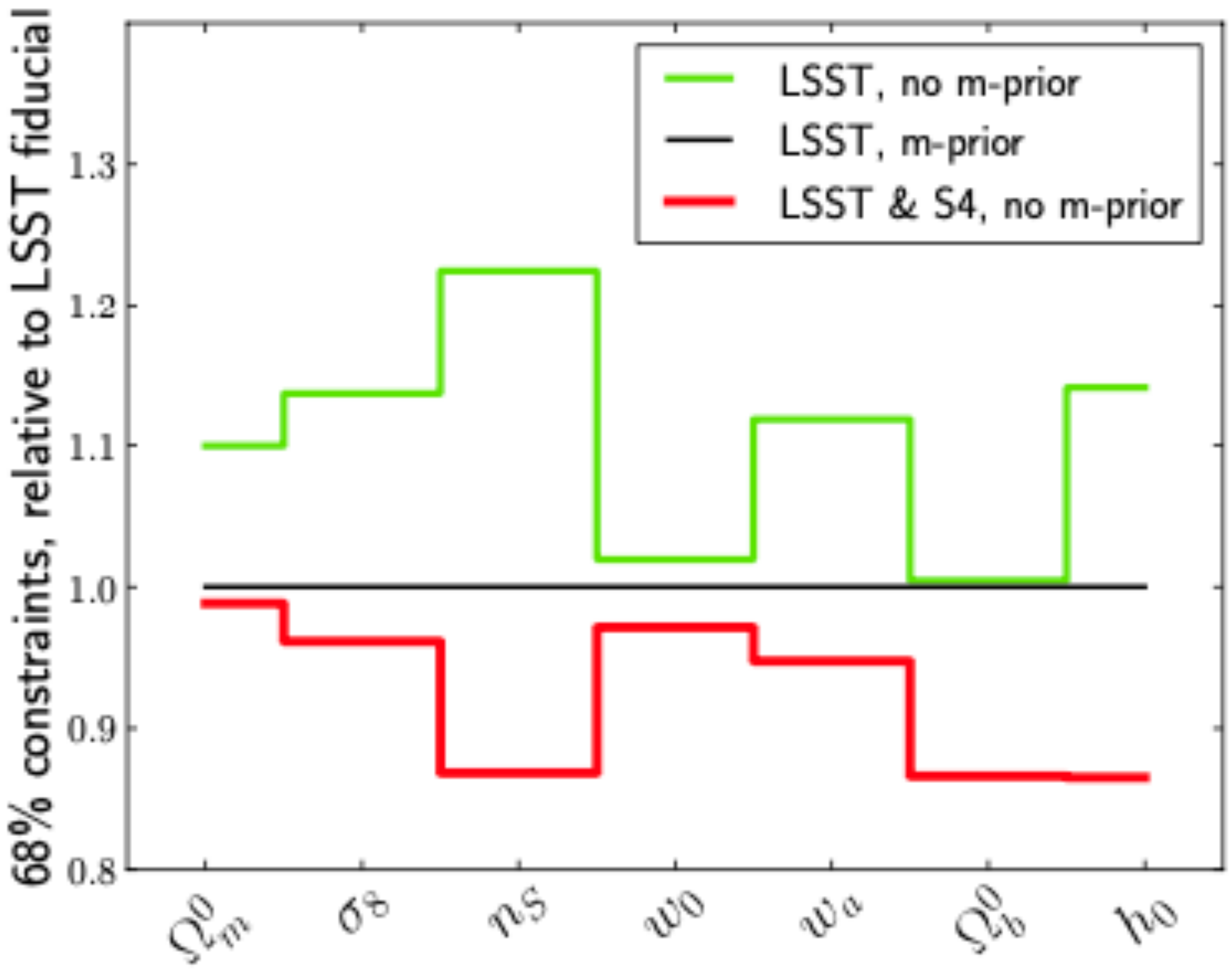
EUCLID REDSHIFT DISTRIBUTIONS

- Strong overlap between the CMB lensing and efficiency and the galaxy distribution of Euclid

Euclid Preparation XV: Forecasting cosmological constraints for the *Euclid* and CMB joint analysis



CALIBRATING OF WEAK LENSING SURVEYS



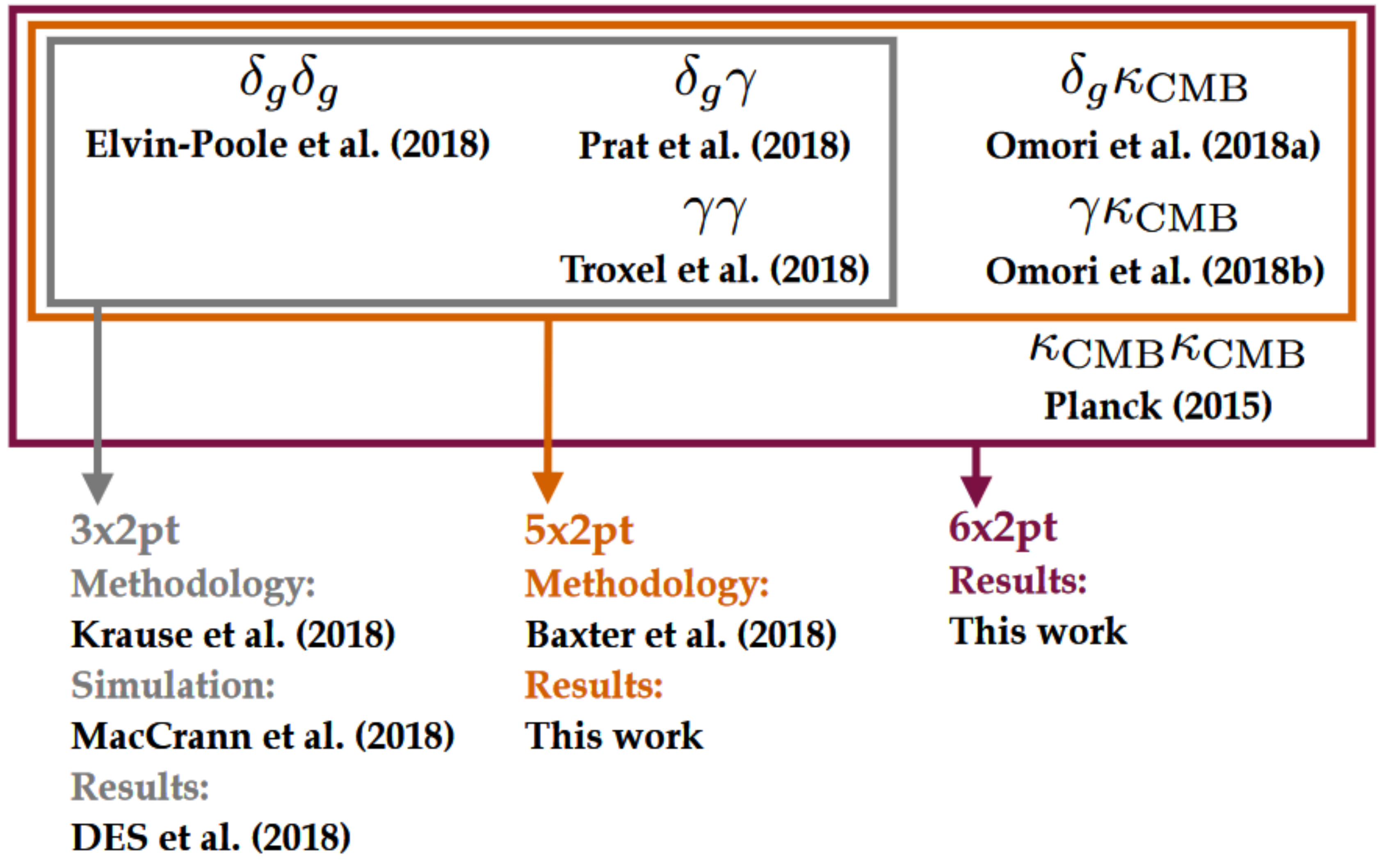
Schaan et al. 2017

- Calibrating magnification bias of LSST with the CMB lensing of CMB-S4

EUCLID CROSS CMB LENSING DATA VECTOR

$$\ln L \propto (\hat{C}_\ell - C_\ell^{\rm th})^{\rm T} {\rm Cov}^{-1} (\hat{C}_\ell - C_\ell^{\rm th})$$

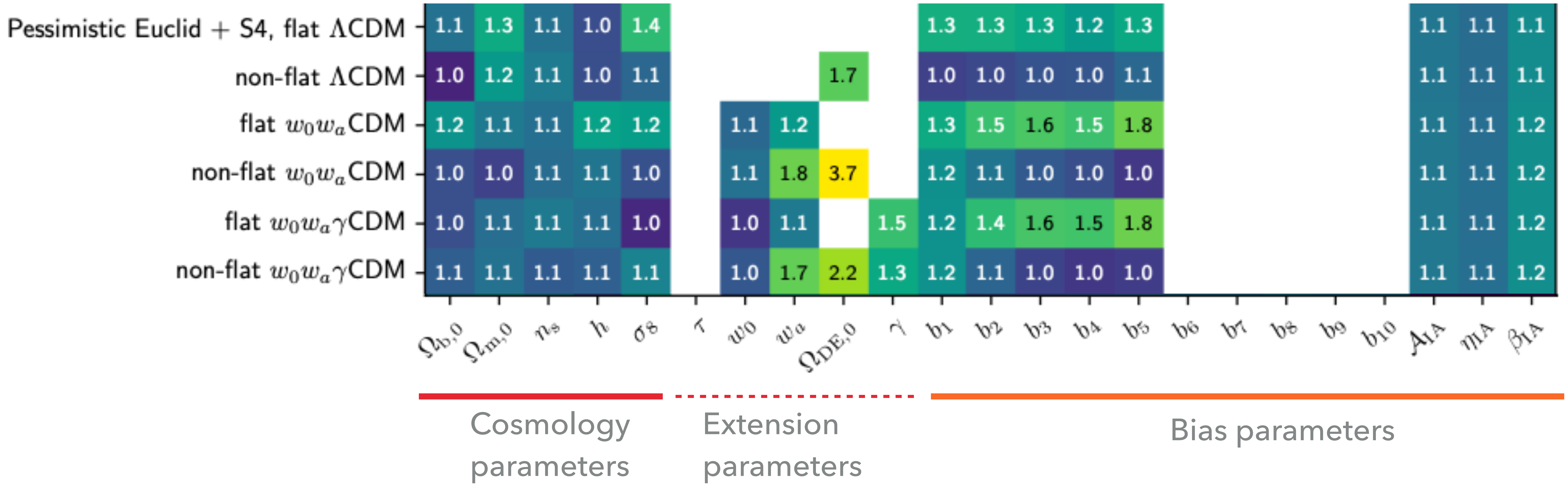
$$\vec{O}_{XC}(\ell) = \{C_\ell^{\kappa_{\rm CMB},\kappa_{\rm CMB}}, C_\ell^{\kappa_{\rm CMB},{\rm GCph_i}}, C_\ell^{\kappa_{\rm CMB},{\rm WL_i}}, C_\ell^{\rm GCph_i,GCph_j}, C_\ell^{\rm WL_i,WL_j}, C_\ell^{\rm WL_i,GCph_j}\}$$



DES x SPT data vector

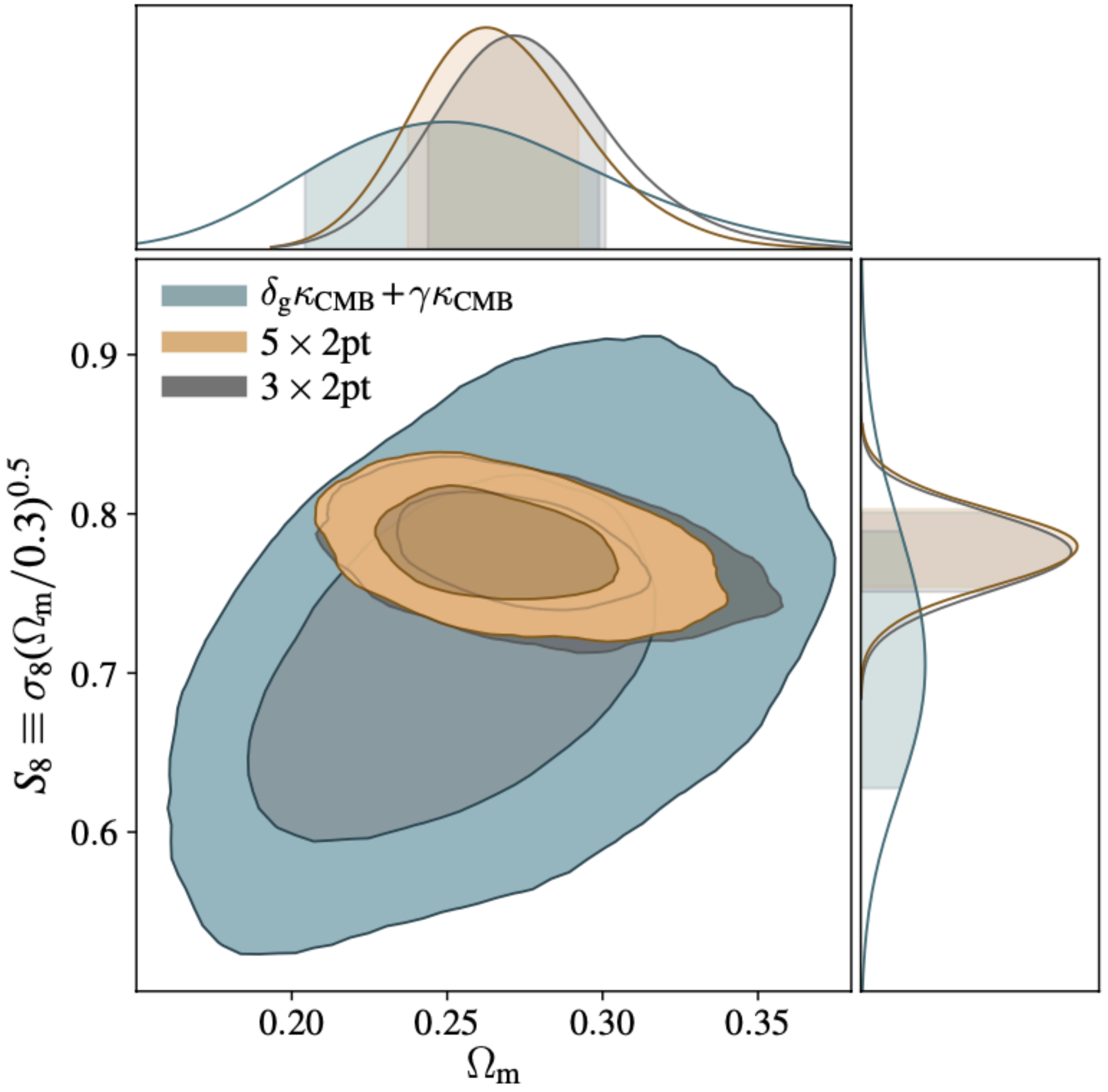
	$C_\ell^{\kappa_{\rm CMB},\kappa_{\rm CMB}}$	$C_\ell^{\kappa_{\rm CMB},{\rm GCph_i}}$	$C_\ell^{\kappa_{\rm CMB},{\rm WL_j}}$	$C_\ell^{\rm GCph_i,GCph_j}$	$C_\ell^{\rm WL_i,WL_j}$	$C_\ell^{\rm WL_i,GCph_j}$
$C_\ell^{\kappa_{\rm CMB},\kappa_{\rm CMB}}$	Cov(kk, kk)	Cov(kk, k-GC$_i$)	Cov(kk, k-WL$_j$)	Cov(kk, GC$_i$-GC$_j$)	Cov(kk, WL$_i$-WL$_j$)	Cov(kk, WL$_i$-GC$_j$)
$C_\ell^{\kappa_{\rm CMB},{\rm GCph_i}}$		Cov(k-GC$_j$, k-GC$_i$)	Cov(k-GC$_i$, k-WL$_j$)	Cov(k-GC$_i$,GC$_j$-GC$_k$)	Cov(k-GC$_i$,WL$_j$-WL$_k$)	Cov(k-GC$_i$,WL$_j$-GC$_k$)
$C_\ell^{\kappa_{\rm CMB},{\rm WL_j}}$			Cov(k-WL$_i$, k-WL$_j$)	Cov(k-WL$_i$, GC$_i$, GC$_j$)	Cov(k-WL$_i$, WL$_i$-WL$_j$)	Cov(k-WL$_i$, WL$_i$-GC$_j$)
$C_\ell^{\rm GCph_i,GCph_j}$				EUCLID 3X2pt COVARIANCE MATRIX		
$C_\ell^{\rm WL_i,WL_j}$	Cov(A B, A' B') $= \frac{\delta^{\rm K}_{\ell\ell'}}{(2\ell+1)}\left[\Delta C_{ik}^{AA'}(\ell)\Delta C_{jl}^{BB'}(\ell') + \Delta C_{im}^{AB'}(\ell)\Delta C_{jk}^{BA'}(\ell')\right]$			$[N_l\ N_z\ (2N_z+1)]$ X $[N_l\ N_z\ (2N_z+1)]$		
$C_\ell^{\rm WL_i,GCph_j}$	$\Delta C_{ij}^{AB}(\ell) = \frac{1}{\sqrt{f_{\rm sky}\Delta\ell}}\left[C_{ij}^{AB}(\ell) + N_{ij}^{AB}(\ell)\right]$			4200 X 4200		

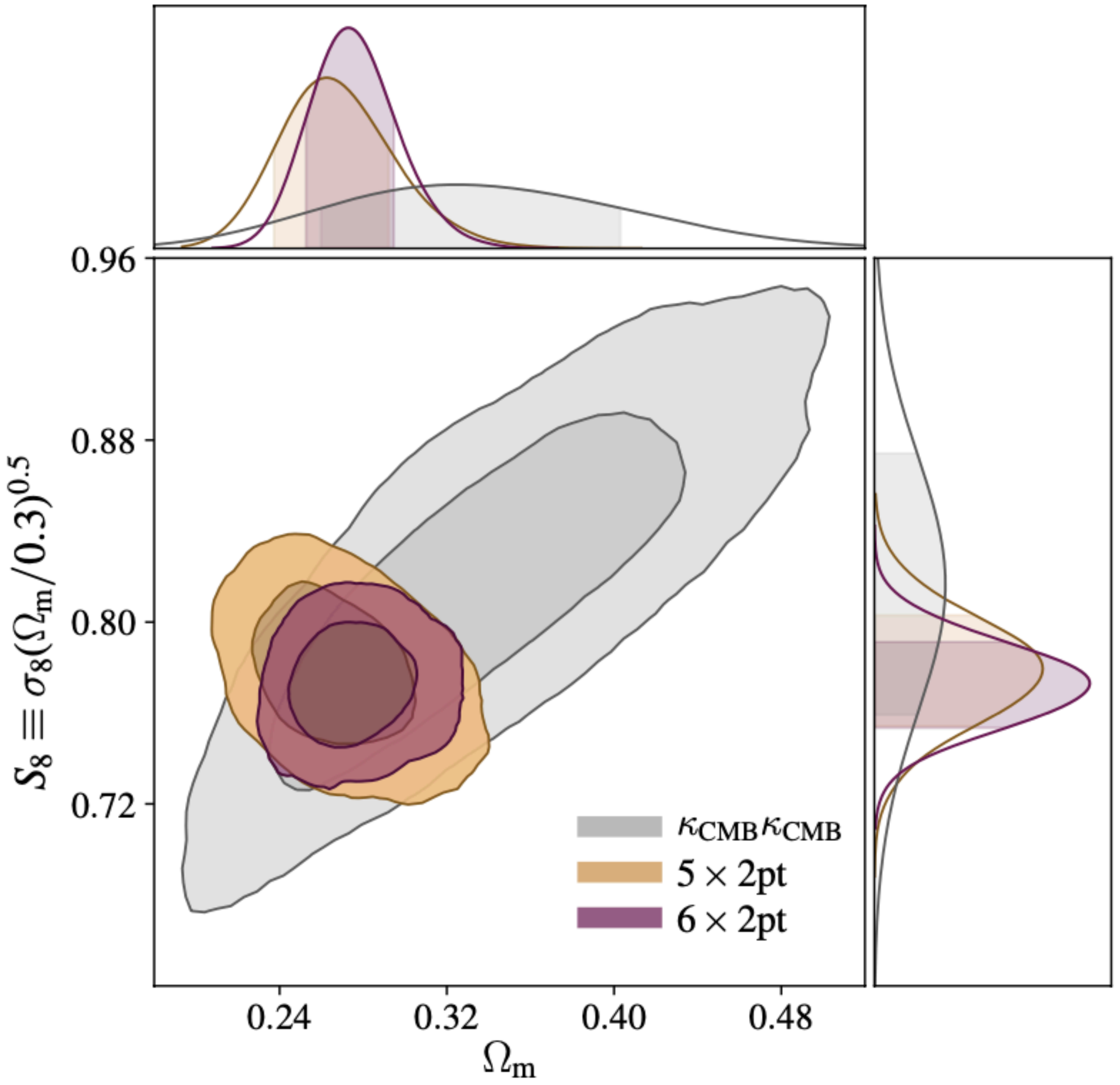
EUCLID FORECASTS



- For a standard LCDM cosmology, adding CMB lensing and cross correlation with Euclid improves constraints on galaxy bias by 30%, and in intrinsic alignment bias by 10%

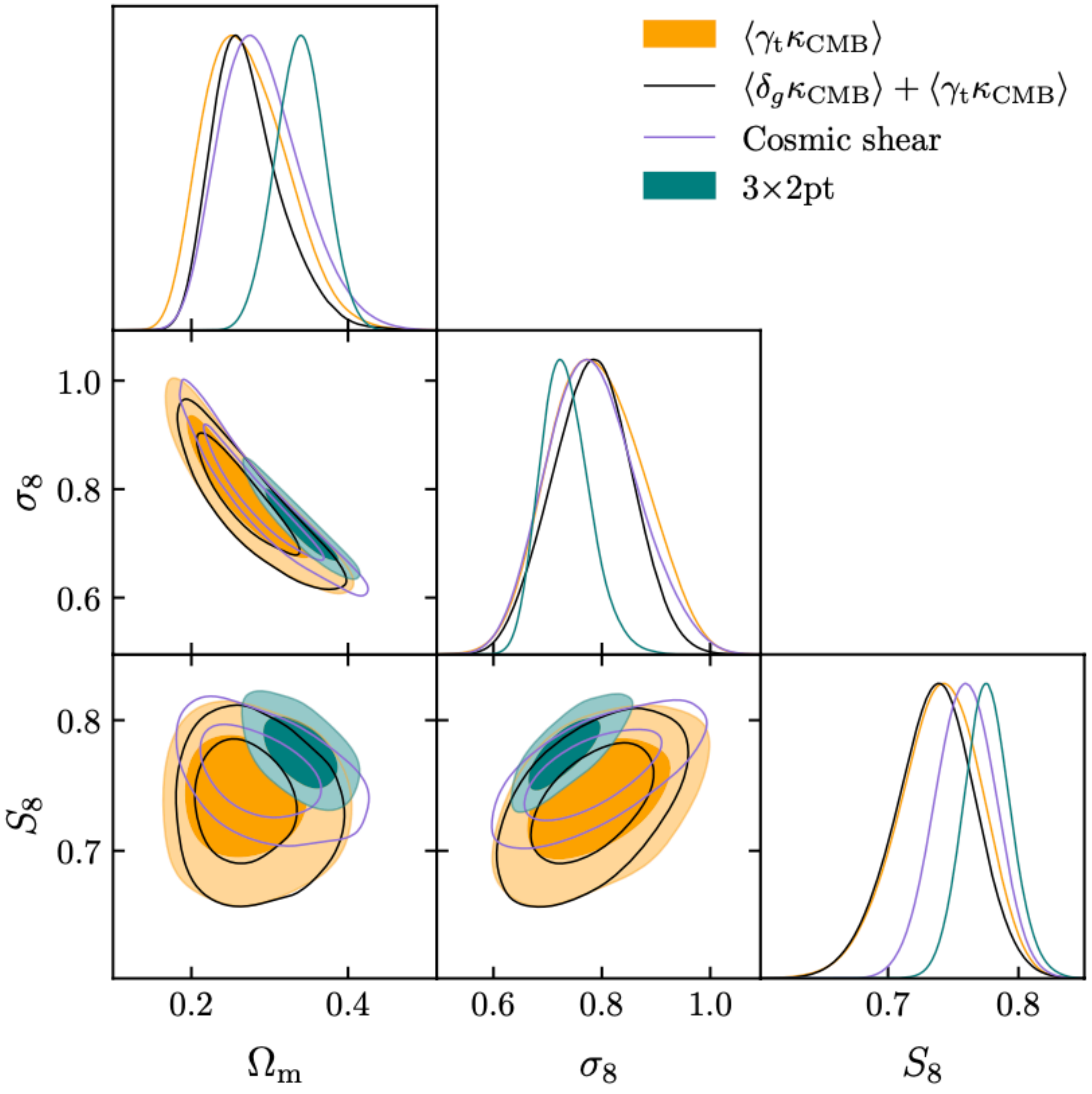
DES X SPT RESULTS





Abbott et al 2018

DES X SPT RESULTS



Chang et al 2022

OUTLINE

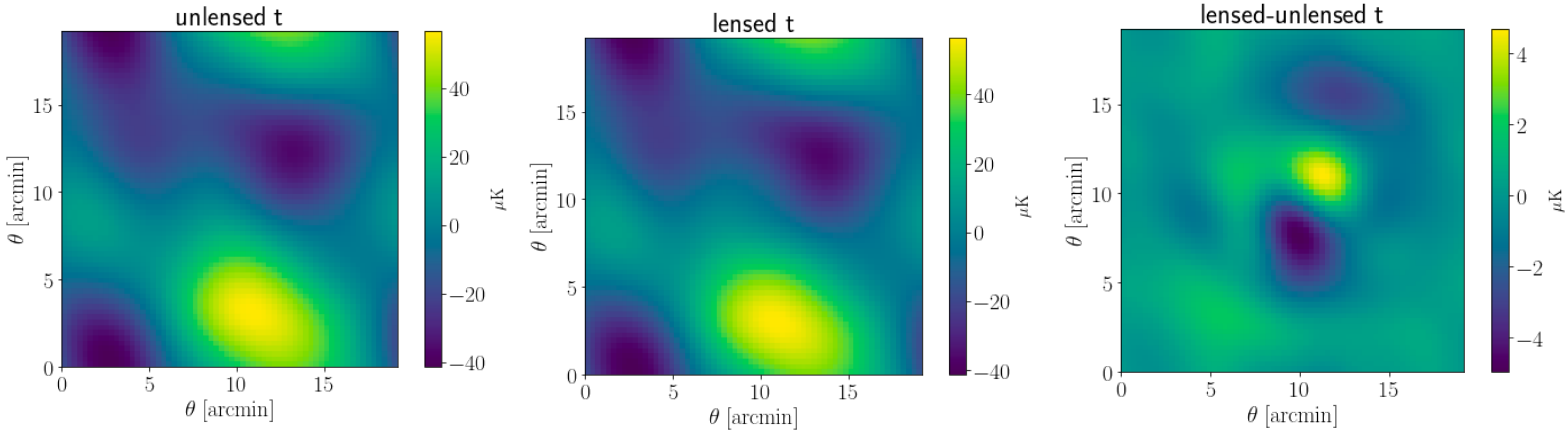
- What is the gravitational lensing of the CMB ?
- How do we measure it?
- Cross correlations with galaxy surveys
- Galaxy clusters and CMB lensing
- Next generation estimators of CMB lensing

GALAXY CLUSTERS

- Most massive collapsed structure in the universe
- Cluster mass function sensitive to $\sigma_8 - \Omega_m$
- Difficult to calibrate the mass (X-Ray, SZ, Galaxy weak lensing)
- CMB lensing being a source at very high redshift it can estimate the weak lensing and thus the mass of all clusters
- However it's still very noisy so can only estimate the mass by stacking groups of clusters

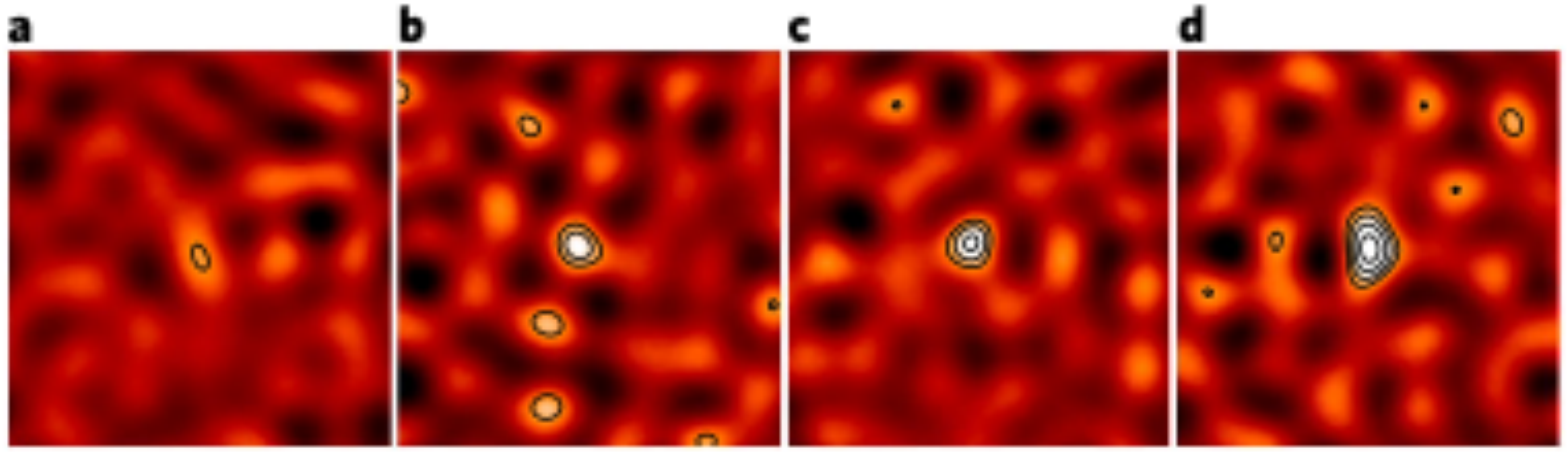
LENSING BY CLUSTERS

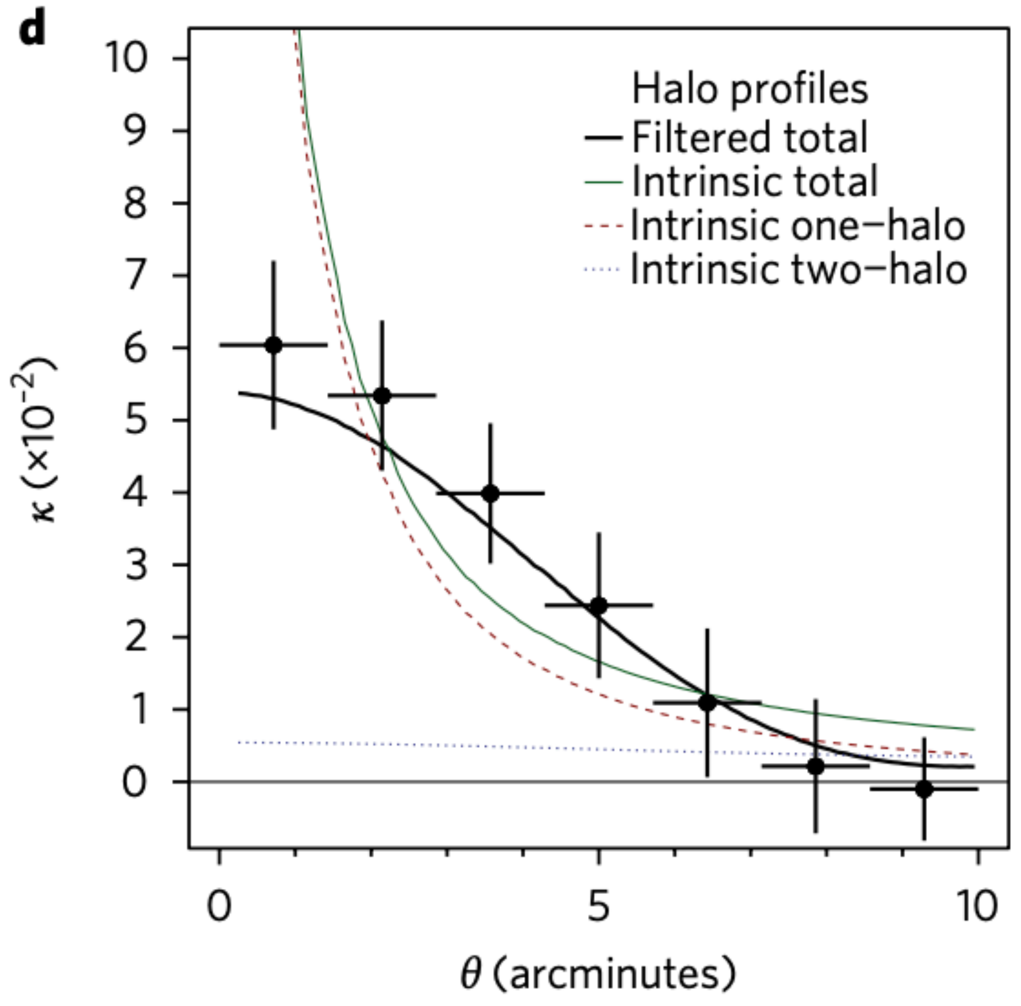
- On small scales the lensing creates a dipolar patern

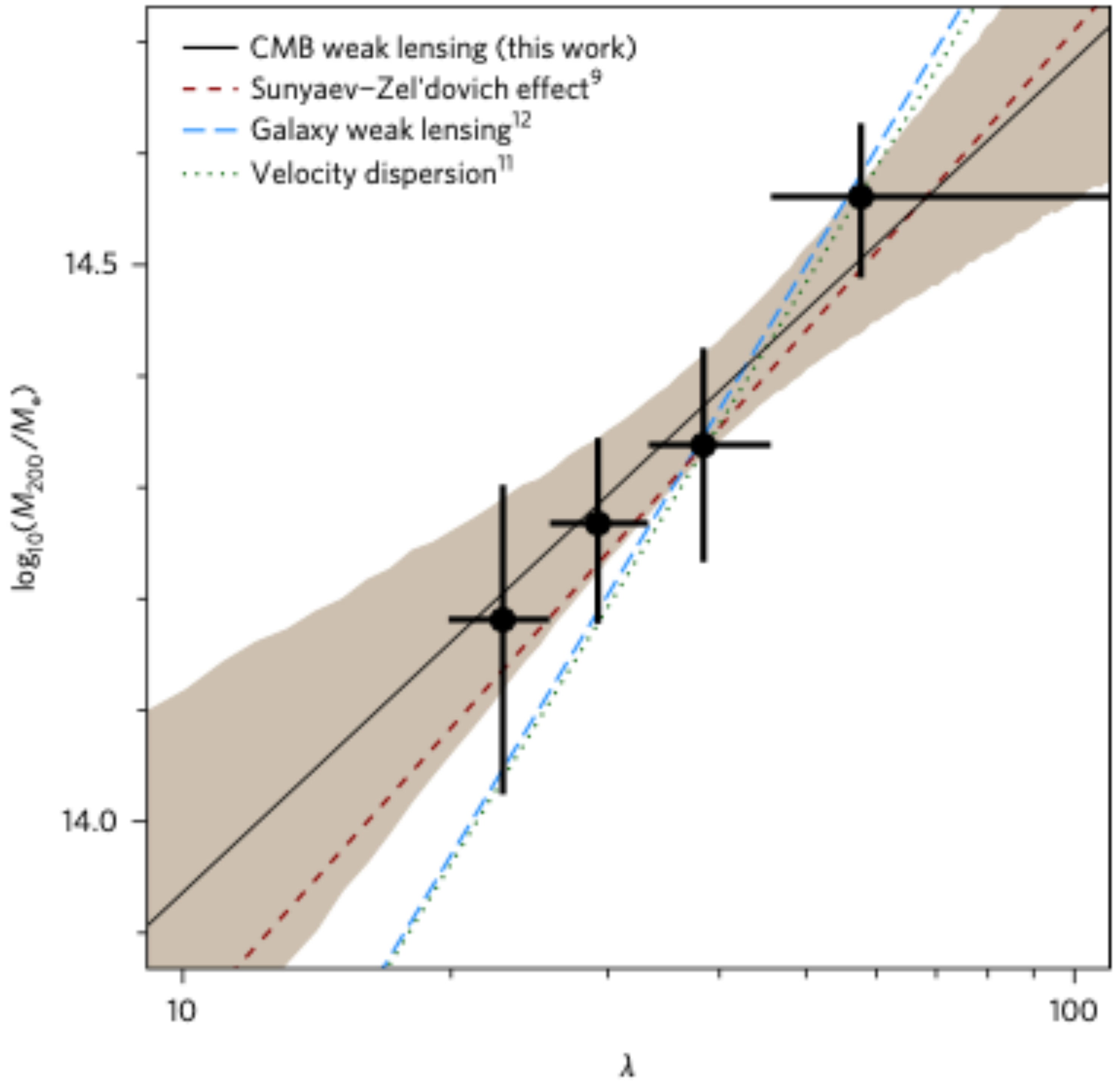


MASS RICHNESS RELATION

- Stacking CMB lensing map at the position of clusters



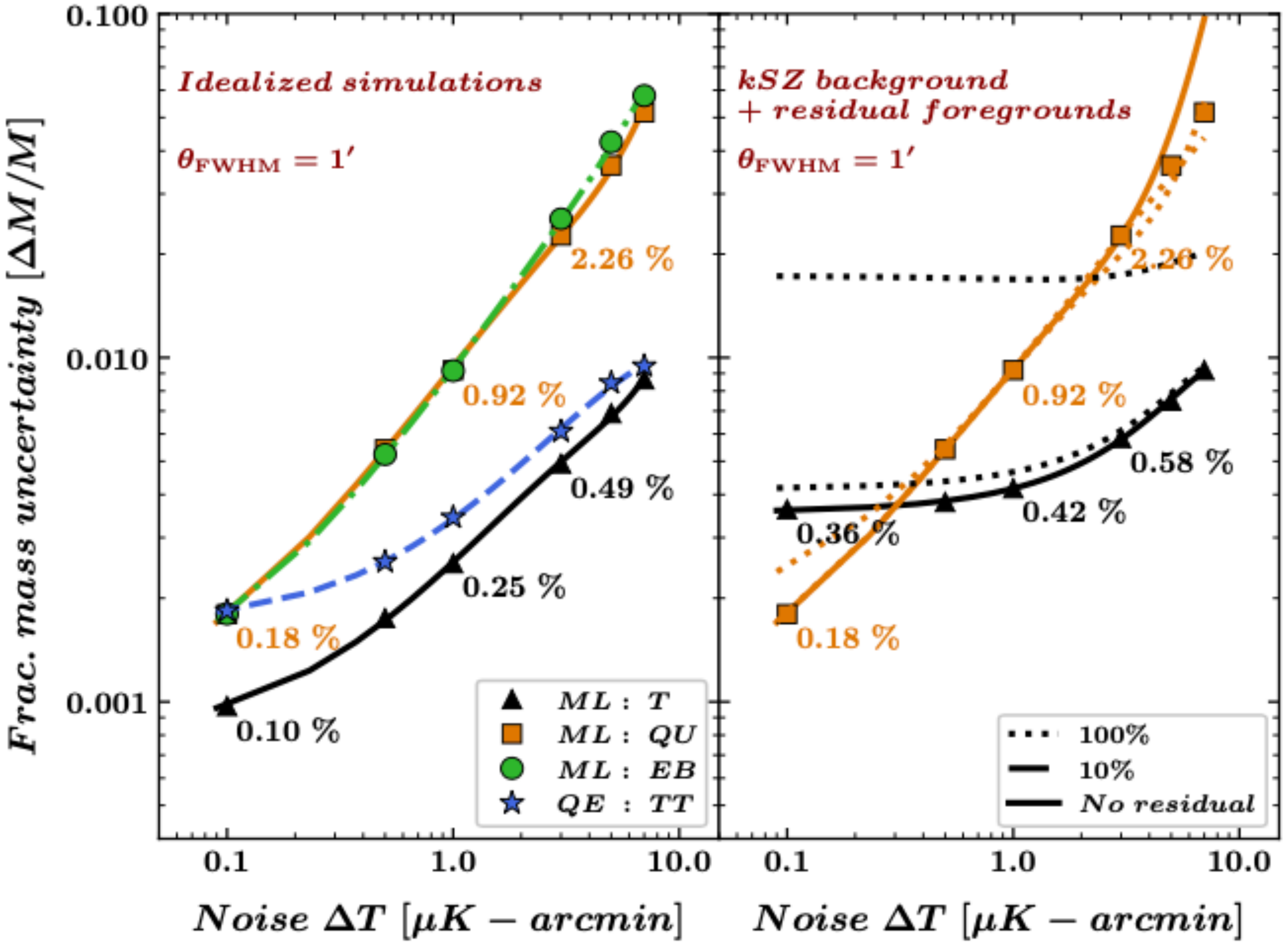




Geach and Peacock 2017

FUTURE CMB SURVEYS

- Will be able to calibrate the mass of 100 000 clusters with sub percent uncertainty
- But very sensitive to SZ signal which contaminates the lensing reconstruction
- Polarisation is expected to be much cleaner



Raghunatan et al 2017

OUTLINE

- What is the gravitational lensing of the CMB ?
- How do we measure it?
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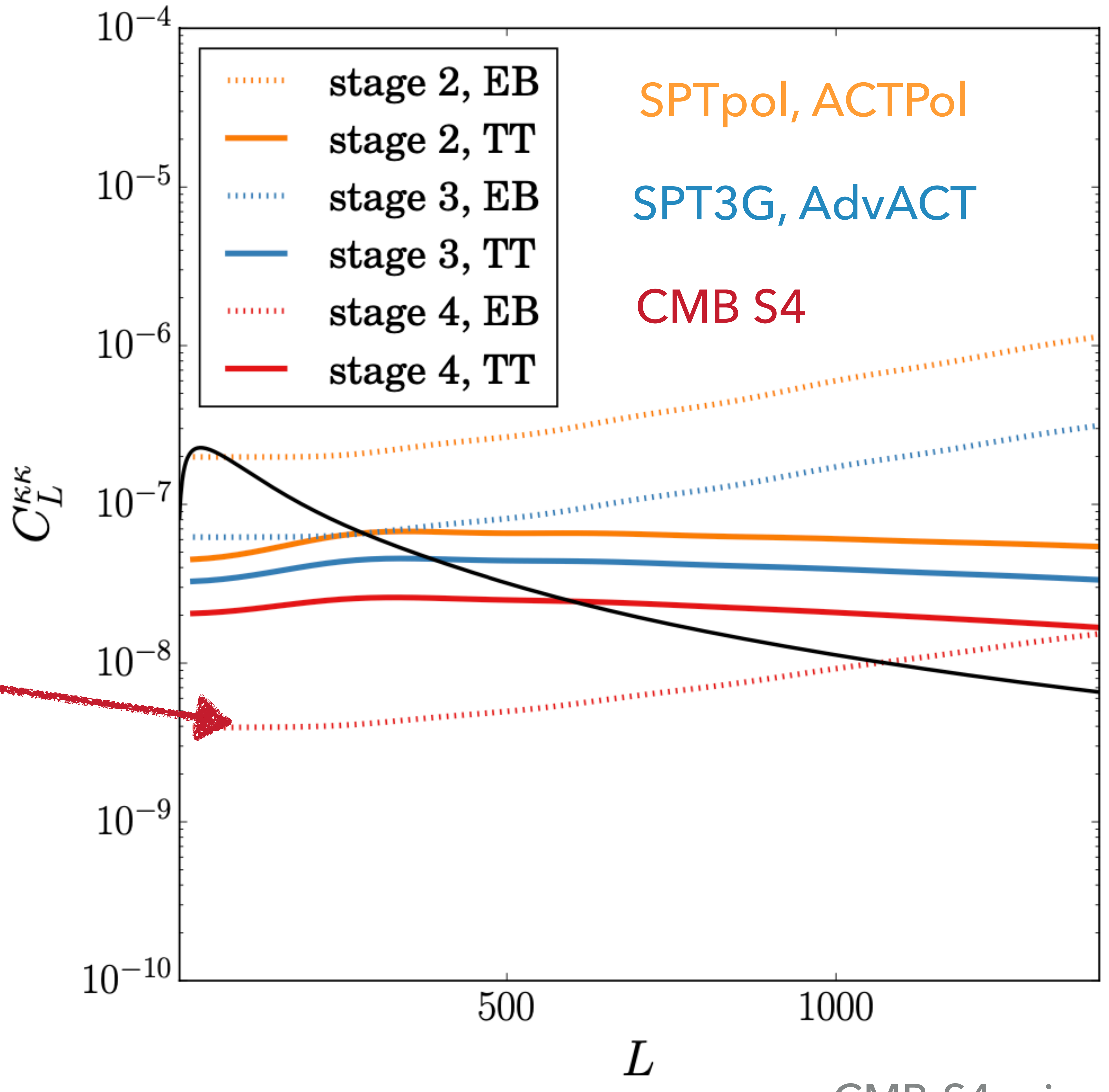
- Next generation estimators of CMB lensing

NEXT GENERATION CMB SURVEYS

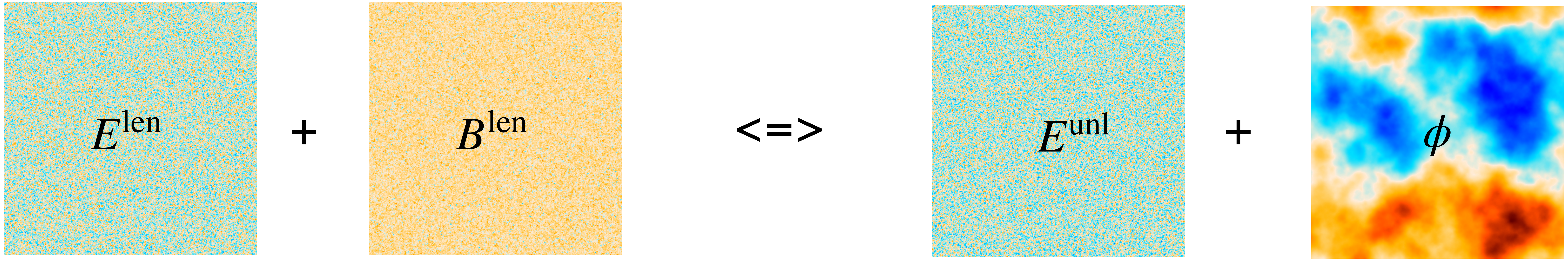
EB (polarisation) estimator will be dominant for CMB S4



CMB-S4 science book 2016

MORE OPTIMAL ESTIMATORS

- Neglecting primordial B modes, one could reconstruct perfectly the lensing field

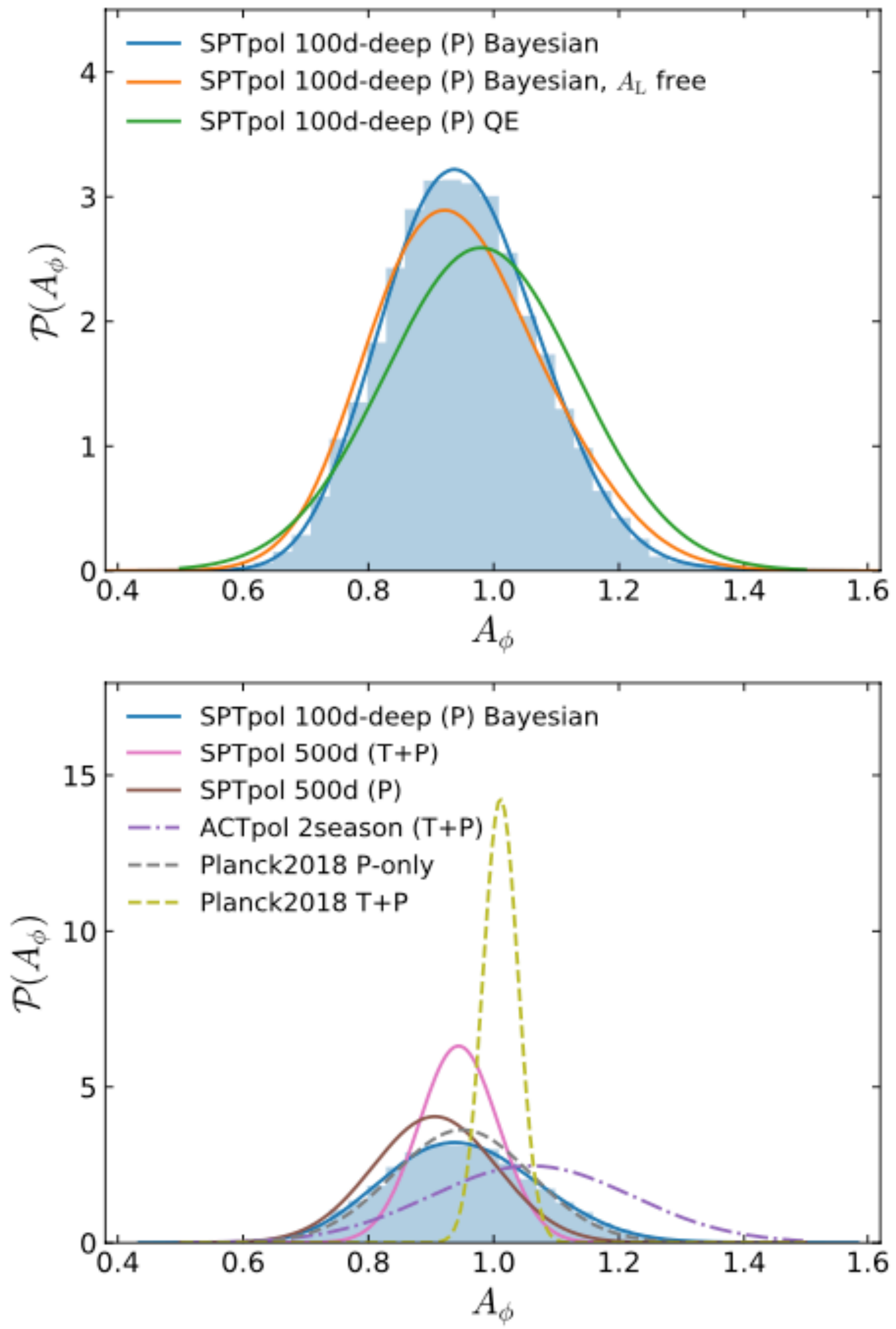


- Likelihood based approach, first introduced in Hirata & Seljak 2003 $P(\phi\,|\,\mathrm{data})$
- How can we find the maximum of this likelihood ?
  - Sampling-based approach -> Millea et al 2020
  - Iterative approach -> Carron & Lewis 2017

BAYESIAN SAMPLING

$$\mathcal{P}\left(\, f, \phi, A_\phi, A_f, P_{\rm cal}, \psi_{\rm pol}, \epsilon_{\rm Q}, \epsilon_{\rm U}, \beta_i \,|\, d \,\right)$$

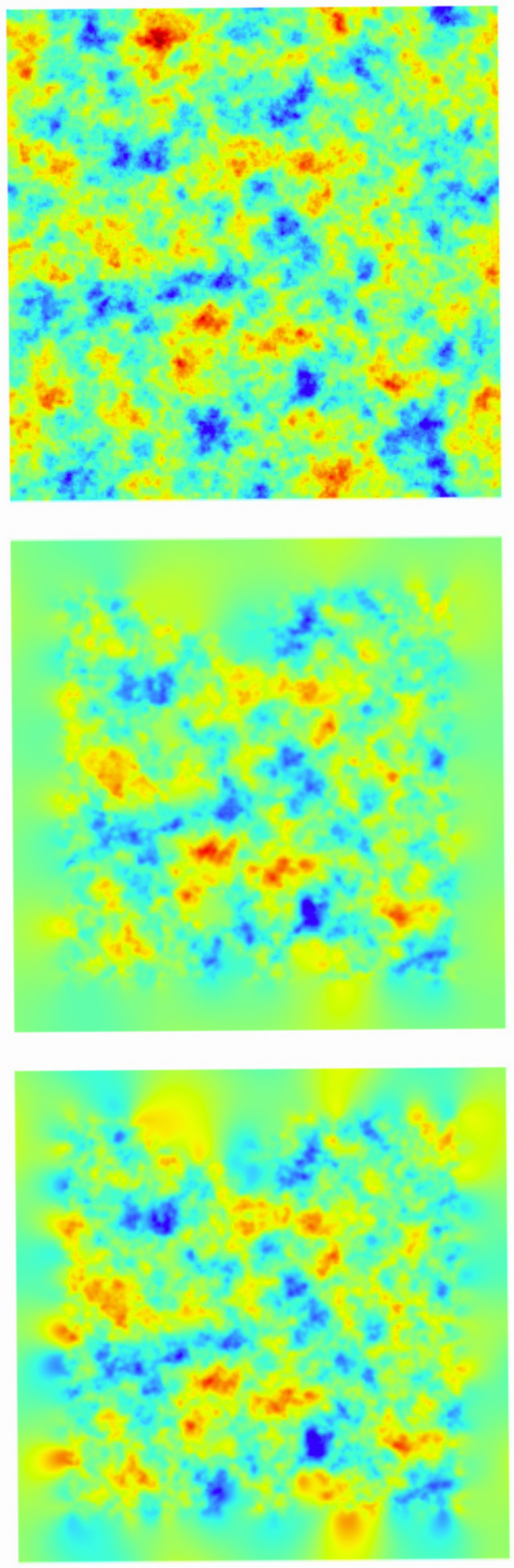
- Sampling with a Monte-Carlo algorithm
- 202,800 free parameters for the 100 deg2 SPT-Pol data of 260 pixels of a side
- 17% improvement for SPT-Pol on the uncertainty of lensing amplitude compared to QE



Millea et al 2020

ITERATIVE APPROACH

- Newton-Raphson iterations on the likelihood
- At each step we get an estimate of the maximum a posteriori lensing field, obtained with a QE
- In practice at each step:
  - delens the data using the deflection estimate
  - apply a quadratic estimator on the resulting maps
  - start again until convergence
- Advantage: fast and based on a well known tool, the quadratic estimator



Input phi

QE

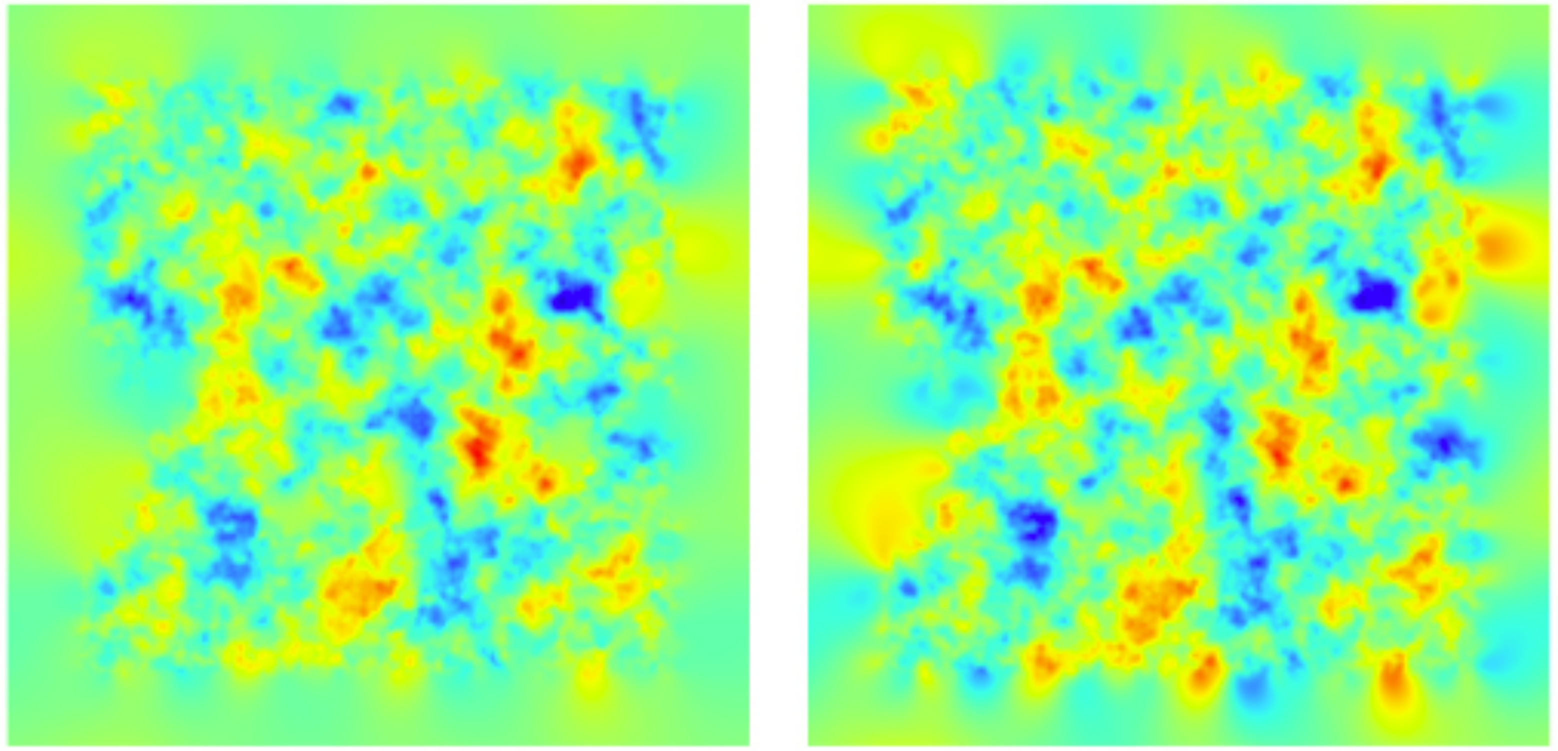
MAP

Carron&Lewis 2017

OPTIMAL LENSING POWER SPECTRUM ESTIMATION

- Problem: cannot track analytically the 4 point function of the lensing power spectrum
- How do we debias the spectrum obtained from the iterative lensing reconstruction ?

QE



MAP

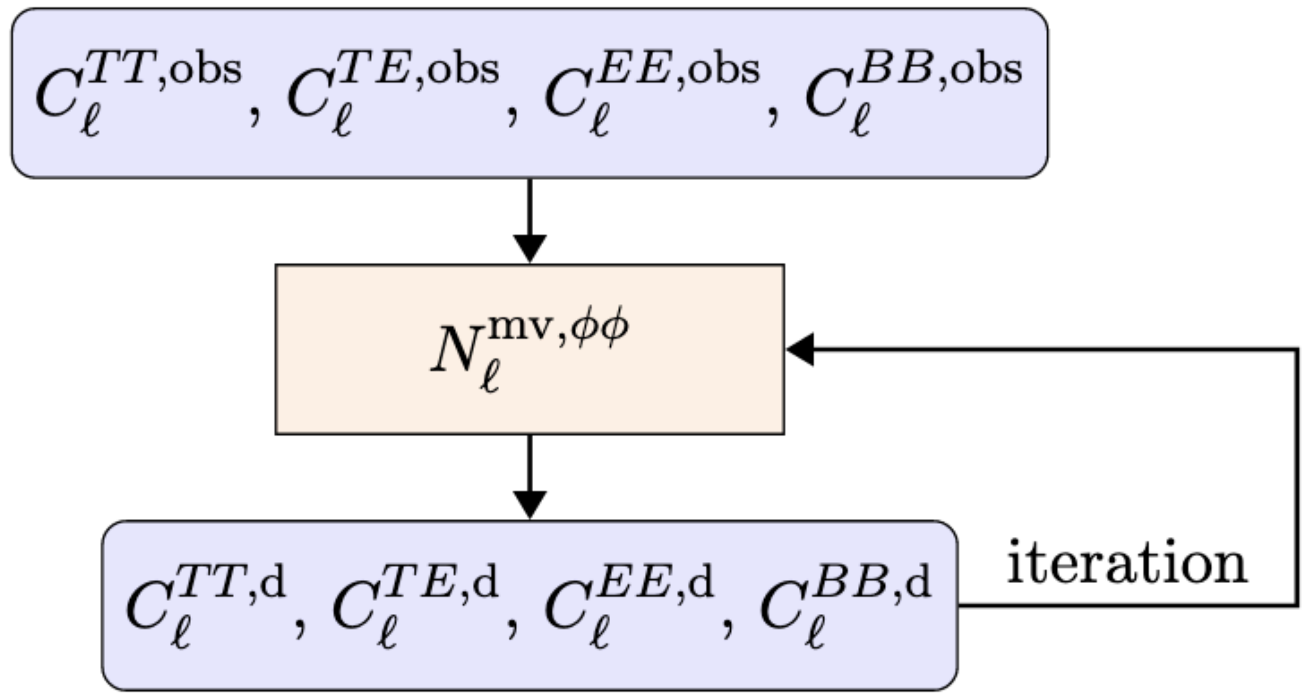
$$C_L^{\hat\phi\hat\phi} = C_L^{\phi\phi} + N_L^0 + N_L^1 + \dots$$

$$C_L^{\hat\phi\hat\phi} = C_L^{\phi\phi} + N_L^{0,\mathrm{MAP}} + N_L^{1,\mathrm{MAP}} + \dots$$

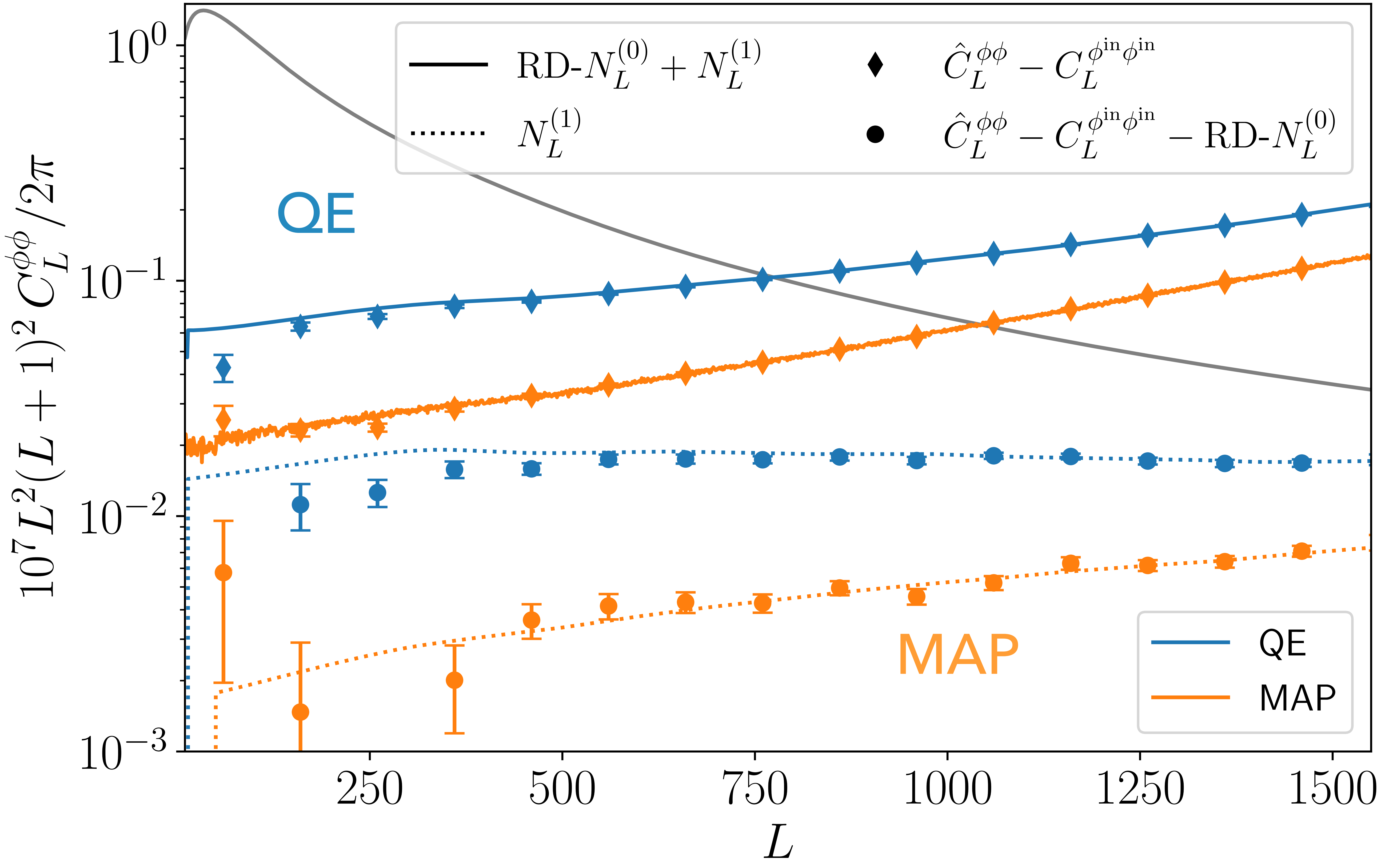
ITERATIVE BIASES

$$C_L^{\hat\phi\hat\phi} = C_L^{\phi\phi} + N_L^0 + N_L^1$$

- Assume N0 and N1 biases are same expression of the QE but with partially delensed CMB spectra, obtained iteratively



Hotinli et al 2021



LL & Carron 2021

CONCLUSION

- CMB lensing is the deflection of the primary image of the CMB by large scale structure
- It creates statistical anisotropies, which can be used to reconstruct the lensing field with a quadratic estimator
- It is sensitive to $\sigma_8 - \Omega_\mathrm{m}$, and to the neutrino mass when combined to BAO and primary CMB.
- Cross correlations increase the cosmological constraints and decrease the importance of systematic uncertainties, and is especially useful in extended LCDM models

Let’s now jump in the pool with a hands on session

Click here: